MVE515 Computational Mathematics - Re-exam

Date: Tuesday, 21 August 2018
Time: 14.00-18.00

- Telephone contact during the exam: Olof Elias, ext 5325.
- The final exam is worth 50 points. 20 points are required for a pass grade 3, 30 points for grade 4, and 40 points for grade 5. The bonus points gathered during the course is added to your exam total.
- Aid: you may use the supplied formula sheet.

Problem 1. Solve the following boundary value problem by integrating twice:

$$\begin{cases} -\mathrm{D}((2-x)\,\mathrm{D}u) = x^2 \quad \text{in } (1, \dfrac{3}{2}), \\ u(1) = 1, \quad \mathrm{D}u(\dfrac{3}{2}) = 0. \end{cases}$$

(7p)

Problem 2. Evaluate the integral

$$\int_0^1 \int_x^1 \cos(\frac{x}{y})\,\mathrm{d}y\mathrm{d}x.$$

(7p)

Problem 3. A particle starts at the point (1,0) and moves along the x-axis to (-1,0), then along to the semicircle $y = \sqrt{1-x^2}$ to the starting point. Use Green's Theorem to find the work done on this particle by the force field

$$\mathbf{F}(x, y) = \langle 2x, x^2 + 3xy^2 \rangle.$$

(7p)

Problem 4. Verify that Stokes' Theorem is true for the vector field $\mathbf{F}(x, y, z) = y^2 z\mathbf{i} - 2y\mathbf{j} + x\mathbf{k}$ where S is the part of the paraboloid $x^2 + y^2 = 4z$ that lies below the plane $z = 1$ and is oriented upward. (8p)

Problem 5. Evaluate

$$\iiint_E z\,\mathrm{d}V,$$

where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$ and $x+y+z = 1$. (8p)

Please turn over!

Problem 6.

(a) Write down the boundary value problem for the 2D stationary heat equation on the triangle vertices with $P_1 = (0,0)$, $P_2 = (1,0)$, $P_3 = (1,1)$, with heat conductivity equals $1 + x + y$, constant heat source density equals 2, and constant ambient temperature equals 1 on all sides. On the boundary $x = 1$ the heat transfer coefficient equals 2, on the boundary $y = 0$ the heat transfer coefficient equals 1, while on the rest of the boundary it equals 0. There is no prescribed heat influx at the boundary. (4p)

(b) Write down the weak formulation of the problem. (4p)

(c) Write down the finite element basis functions ϕ_1, ϕ_2, ϕ_3 for a triangulation that consists of a single triangle $T = \Omega$ with nodes $P_1 = (0,0)$, $P_2 = (1,0)$, $P_3 = (1,1)$. (3p)

(d) Compute the element

$$b_2 = \iint_{\Omega} f \phi_2 \,\mathrm{d}A + \int_{\Gamma} (g + \kappa u_A) \phi_2 \,\mathrm{d}s.$$

of the load vector. (2p)