Problem set 1.

Problem 1.1. Write the weak formulation of the boundary value problem:

$$\begin{cases} -u'' + bu = f \quad \text{in } (0, L), \\ u'(0) = g_0, \quad u(L) = u_L. \end{cases}$$

Problem 1.2. Write the weak formulation of the initial-boundary value problem:

$$\begin{cases} D_t u(x,t) - D_x\big((x-1)D_x u(x,t)\big) = xt \quad \text{in } (0,2),\ t > 0, \\ -D_x u(0,t) + u(0,t) = 3, \quad u(2,t) = 4 \qquad\qquad\qquad\quad t > 0, \\ u(x,0) = \cos x \qquad\qquad\qquad\qquad\qquad\quad \text{in } (0,2). \end{cases}$$

Problem 1.3. Solve the following boundary value problem by integrating twice:

$$\begin{cases} -\mathrm{D}((3-x)\,\mathrm{D}u) = 3x^2 + 2x \quad \text{in } (0,1), \\ u'(0) = 11, \quad u(1) = \ln 8. \end{cases}$$

Problem 1.4. Find the volume of the solid that lies above the rectangle $R = [0,1] \times [0,1]$ and below the surface $z = \frac{x}{1+xy}$.

Problem 1.5. Evaluate the integral

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\,\mathrm{d}x\mathrm{d}y.$$

Hint: See Example 5 in Section 15.3.