Problem set 2.

Problem 2.1. Find the volume of the solid that is bounded by the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ ($a > 0$ is a constant).

Problem 2.2. Find the volume of the solid under the surface $f(x, y) = \sqrt{\frac{y}{x}} + \sqrt{xy}$ and above the region R, that is in the first quadrant of the xy-plane bounded by the hyperbolas $xy = 1,\ xy = 9$ and the lines $y = x,\ y = 4x$.
Hint: To evaluate the integral use the transformation $x = \frac{u}{v},\ y = uv,\ u > 0,\ v > 0$.

Problem 2.3. Evaluate the integral

$$\iint_R \sin(9x^2 + 4y^2)\ \mathrm{d}A$$

by making an appropriate change of variables. The region R is the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.

Problem 2.4. An object with mass m moves with position function (a, b, c are constant)

$$\mathbf{r}(t) = \langle a \sin t, b \cos t, ct \rangle, \quad 0 \le t \le \frac{\pi}{2}.$$

What is the work done by the force during the time interval?
Hint: $\mathbf{F}(t) = m\mathbf{a}(t)$.

Problem 2.5. If C is a smooth curve given by a vector function $\mathbf{r},\ a \le t \le b$, show that

$$\int_C \mathbf{r} \cdot \mathrm{d}\mathbf{r} = \frac{1}{2}\big[|\mathbf{r}(b)|^2 - |\mathbf{r}(a)|^2\big].$$