Problem set 3.

Problem 3.1. consider the force field $\mathbf{F}(\mathbf{X}) = \frac{c\mathbf{X}}{|\mathbf{X}|^3}$ for some constant c where $\mathbf{X} = \langle x, y, z\rangle$. Find the work done by $\mathbf{F}$ in moving an object from a point P_1 (with distance d_1 from the origin) to a point P_2 (with distance d_2 from the origin), in terms of d_1 and d_2.
Hint: Find the potential function f such that $\mathbf{F} = \nabla f$, using section 16.1.

Problem 3.2. Verify that Green's Theorem is true for the vector field $\mathbf{F}(x, y) = \langle xy^2, -x^2y\rangle$ and C consists of the parabola $y = x^2$ from $(-1, 1)$ to $(1, 1)$ and the line segment from $(1, 1)$ to $(-1, 1)$.

Problem 3.3. Determine if the vector field

$$\mathbf{F}(x, y, z) = \left\langle e^x \sin(yz), ze^x \cos(yz), ye^x \cos(yz)\right\rangle$$

is conservative. If yes, find a function f such that $\mathbf{F} = \nabla f$.

Problem 3.4. Find the area of the paraboloid $x = y^2+z^2$ that lies inside the cylinder $y^2+z^2 = 9$.

Problem 3.5. Find the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for $\mathbf{F}(x, y, z) = \langle y, -x, 2z\rangle$ and the oriented surface S,and S is the hemisphere $x^2 + y^2 + z^2 = 4,\ z \geq 0$, oriented downward. (use the positive outward orientation).