3.1 $f(\vec{x}) = f(x,y,z) = \dfrac{-c}{|\vec{x}|} = -\dfrac{c}{\sqrt{x^2+y^2+z^2}}$ (See section 16.1)

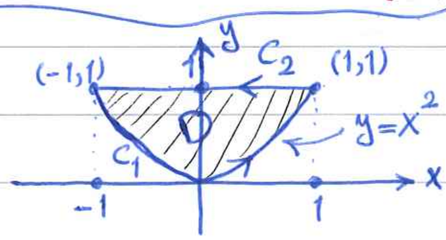
Assume $P_1=(x_1,y_1,z_1)$ and $P_2=(x_2,y_2,z_2)$

Then $$W=\int_C \vec{F}\cdot d\vec{r} = \int_C \nabla f\cdot d\vec{r} = f(P_2)-f(P_1) = \frac{-c}{|\vec{OP_2}|} - \left(-\frac{c}{|\vec{OP_1}|}\right) = \boxed{c\left(\frac{1}{d_1}-\frac{1}{d_2}\right)}$$

3.2

$C_1:\ x=x,\ y=x^2,\quad -1\leq x\leq 1$

$C_2:\ x=-x,\ y=1,\quad -1\leq x\leq 1$



We should show that
$$\int_C \vec{F}\cdot d\vec{r} = \iint_D (Q_x-P_y)\,dA$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_{C_1}\vec{F}\cdot d\vec{r} + \int_{C_2}\vec{F}\cdot d\vec{r} = \int_{-1}^{1}\langle x(x^2)^2, -x^2(x^2)\rangle\cdot\langle 1, 2x\rangle\,dx$$
$$+\int_{-1}^{1}\langle (-x)(1)^2, -(-x^2)(1)\rangle\cdot\langle -1, 0\rangle\,dx$$
$$= \int_{-1}^{1} -x^5\,dx + \int_{-1}^{1} x\,dx = 0$$

$$\iint_D (Q_x-P_y)\,dA = \iint_D \left[\frac{\partial}{\partial x}(-x^2y) - \frac{\partial}{\partial y}(xy^2)\right]dA = \iint_D (-2xy-2xy)\,dA$$
$$= \int_{-1}^{1}\int_{x^2}^{1}(-4xy)\,dy\,dx = \int_{-1}^{1}(-2x+2x^5)\,dx = 0$$

3.3
$$\text{Curl}\,\vec{F} = \nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y\rangle$$
$$= \Big\langle \left(e^x\cos yz - zye^x\sin yz\right) - \left(e^x\cos yz - yze^x\sin yz\right),$$
$$\left(ye^x\cos yz\right) - \left(ye^x\cos yz\right), \left(ze^x\cos yz\right) - \left(ze^x\cos yz\right)\Big\rangle$$
$$= \langle 0,0,0\rangle$$

So $\vec{F}$ is conservative.

$$\begin{cases} f_x = e^x\sin yz \\ f_y = ze^x\cos yz \\ f_z = ye^x\cos yz \end{cases} \Longrightarrow \begin{cases} f = e^x\sin yz + g(y,z) \\ f_y = ze^x\cos yz \end{cases} \Rightarrow \begin{cases} f_y = ze^x\cos yz + g_y(y,z) \\ f_y = ze^x\cos yz \end{cases}$$

$\Rightarrow g_y(y,z)=0 \Rightarrow g(y,z)=h(z)$

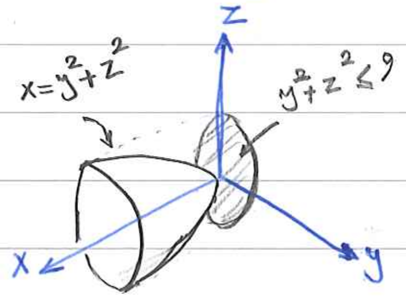
Hence $\begin{cases} f=e^x \sin yz + h(z) \\ f_z = ye^x \cos yz \end{cases} \Rightarrow \begin{cases} f_z = ye^x \cos yz + h'(z) \\ f_z = ye^x \cos yz \end{cases} \Rightarrow h'(z)=0 \Rightarrow h(z)=K$

Therefor

$$\boxed{f(x,y,z)=e^x \sin yz + K}$$

3.4 A parametric representation of the surface is

$x=y^2+z^2, \quad y=y, \quad z=z, \quad \underbrace{0 \le y^2+z^2 \le 9}_{D}$



$\vec{r}_y \times \vec{r}_z = \langle 2y,1,0\rangle \times \langle 2z,0,1\rangle$

$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2y & 1 & 0 \\ 2z & 0 & 1 \end{vmatrix} = \langle 1,-2y,-2z\rangle$

$A(S) = \iint_D |\vec{r}_y \times \vec{r}_z|\, dA = \iint_D \sqrt{1+4(y^2+z^2)}\, dA = \int_0^{2\pi}\int_0^3 \sqrt{1+4r^2}\, r\, dr\, d\theta$

$= \begin{Bmatrix} u=1+4r^2 \\ du=8r\,dr \end{Bmatrix} = \int_0^{2\pi} d\theta \left[\frac{1}{12}(1+4r^2)^{3/2}\right]_{r=0}^{r=3}$

$$\boxed{= \frac{\pi}{6}(37\sqrt{37}-1)}$$

3.5 $z=g(x,y)=\sqrt{4-x^2-y^2}$ with $D=\{(x,y) \mid x^2+y^2\le 4\}$

So $\vec{r}_x \times \vec{r}_y = \langle -g_x, -g_y, 1\rangle = \left\langle \frac{-x}{\sqrt{4-x^2-y^2}}, \frac{-y}{\sqrt{4-x^2-y^2}}, 1\right\rangle$

S has downward orientation, so

$\iint_S \vec{F}\cdot d\vec{S} = -\iint_D \left[-y\frac{1}{2}(4-x^2-y^2)^{-\frac{1}{2}}(-2x) - (-x)\frac{1}{2}(4-x^2-y^2)^{-\frac{1}{2}}(-2y) + 2z\right] dA$

$= -\iint_D \left[\frac{xy}{\sqrt{4-x^2-y^2}} - \frac{xy}{\sqrt{4-x^2-y^2}} + 2\sqrt{4-x^2-y^2}\right] dA$

$= -2\int_0^{2\pi}\int_0^2 \sqrt{4-r^2}\, r\, dr\, d\theta = \begin{Bmatrix} u=4-r^2 \\ du=-2r\,dr \end{Bmatrix} = -2\int_0^{2\pi} d\theta \left[-\frac{1}{2}\frac{2}{3}(4-r^2)^{\frac{3}{2}}\right]_{r=0}^{r=2}$

$$\boxed{= -\frac{32}{3}\pi}$$