1. Låt ES vara ekvationssystemet $\begin{cases} x + y - 2z = 9 \\ 2x - y + z = 2 \\ x + 2y - z = 9 \end{cases}$.

Bestäm z med hjälp av Cramers regel. (2p)

Lösning:

$$\det A = \begin{vmatrix} 1 & 1 & -2 \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = -8$$

$$\det A_3(b) = \begin{vmatrix} 1 & 1 & 9 \\ 2 & -1 & 2 \\ 1 & 2 & 9 \end{vmatrix} = 1\begin{vmatrix} -1 & 2 \\ 2 & 9 \end{vmatrix} - 1\begin{vmatrix} 2 & 2 \\ 1 & 9 \end{vmatrix} + 9\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = -13-16+45 = 16$$

$$z = \frac{\det A_3(b)}{\det A} = \frac{16}{-8} = -2$$

2. Lös ut X ur matrisekvationen $CXA - B = CX$. Ange speciellt vilka inverser som måste existera. (2p)

Lösning:

$$CXA - B = CX \iff CXA - CX = B \iff$$

$$CX(A-I) = B \iff CX = B(A-I)^{-1} \iff$$

$$X = C^{-1}B(A-I)^{-1} \text{ om } C^{-1}, (A-I)^{-1} \text{ existerar}$$

3. Bestäm inversen till matrisen $\mathbf{A} = \begin{bmatrix} 2 & 3 & -1 \\ 6 & 8 & -2 \\ 2 & 5 & -2 \end{bmatrix}$. (2p)

Lösning:

$$\left[\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 6 & 8 & -2 & 0 & 1 & 0 \\ 2 & 5 & -2 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 3/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & -1 & 1 & -3 & 1 & 0 \\ 0 & 2 & -1 & -1 & 0 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 4 & 3/2 & 0 \\ 0 & 1 & -1 & 3 & -1 & 0 \\ 0 & 0 & 1 & -7 & 2 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1/2 & -1 \\ 0 & 1 & 0 & -4 & 1 & 1 \\ 0 & 0 & 1 & -7 & 2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & -1 \\ -4 & 1 & 1 \\ -7 & 2 & 1 \end{bmatrix}$$

1. Lös ut X ur matrisekvationen $BX - C = BXA$. Ange speciellt vilka inverser som måste existera. (2p)

Lösning:

$$BX - C = BXA \Leftrightarrow BX - BXA = C$$

$$\Leftrightarrow BX(I-A) = C \Leftrightarrow X(I-A)\ B^{-1}C$$

$$\Leftrightarrow X = B^{-1}C(I-A)^{-1} \quad \text{om } \begin{matrix} B^{-1} \\ (I-A)^{-1} \end{matrix} \text{ existerar}$$

2. Låt ES vara ekvationssystemet $\begin{cases} x + y - 2z = 9 \\ 2x - y + z = 2 \\ x + 2y - z = 9 \end{cases}$.

Bestäm y med hjälp av Cramers regel. (2p)

Lösning:

$$\det A_2(b) = \begin{vmatrix} 1 & 9 & -2 \\ 2 & 2 & 1 \\ 1 & 9 & -1 \end{vmatrix} = 1\begin{vmatrix} 2 & 1 \\ 9 & -1 \end{vmatrix} - 9\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 1 & 9 \end{vmatrix}$$

$$= -11 + 27 - 32 = -16$$

$$\det A = \begin{vmatrix} 1 & 1 & -2 \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = 1\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + (-2)\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} =$$

$$= -1 + 3 - 10 = -8 \Rightarrow y = 2$$

3. Bestäm inversen till matrisen $\mathbf{A} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 3 \\ 2 & 1 & -1 \end{bmatrix}$. (2p)

Lösning:

$$\left[\begin{array}{ccc|ccc} -1 & 0 & 1 & 1 & 0 & 0 \\ -2 & 0 & 3 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & +1 & 0 \\ 0 & 1 & 0 & 4 & -1 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -1 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$