Anonym kod	MVE525 Matematisk analys 180404	Sidnr	Poäng
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1. Till nedanstående uppgifter skall korta lösningar redovisas, samt svar anges, på anvisad plats (endast lösningar och svar på detta blad, och på anvisad plats, beaktas).

   (a) Bestäm a så att arean under grafen $f(x) = 1 + a/x^2$ mellan $x = 1$ och $x = 2$ är dubbelt så stor som arean mellan $x = 2$ och $x = 3$. (4p)

   Lösning:

   $$\left[x-\frac{a}{x}\right]_1^2 = 2\left[x-\frac{a}{x}\right]_2^3 \Leftrightarrow 1+\frac{a}{2} = 2\left(1+\frac{a}{6}\right)$$

   $$\Leftrightarrow \frac{a}{2}-\frac{a}{3} = 1 \Leftrightarrow a = 6$$

   Svar: $a = 6$

   (b) Bestäm inflexionspunkter till funktionen $f(x) = \dfrac{x^2}{2} + \dfrac{1}{x}$. Ange intervall där funktionen är uppåt resp nedåt konkav. (dvs konvex/konkav) (3p)

   Lösning: $f'(x) = x - \frac{1}{x^2}$ $\quad f''(x) = 1 + \frac{2}{x^3} = \frac{x^3+2}{x^3} = 0 \Leftrightarrow x = -\sqrt[3]{2}$

   $$f''(-2) = \frac{-6}{-8} \quad f''(-1) = \frac{1}{-1} \quad f''(1) = \frac{3}{1}$$

	$-\sqrt[3]{2}$		0	x
f''	$+$	$-$	$+$	
	CU	CD	CU	

   Svar: $x = -\sqrt[3]{2}$ $\quad$ CU: $x < -\sqrt[3]{2}$ samt $x > 0$ $\quad$ CD: $-\sqrt[3]{2} < x < 0$

   (c) Ange den antiderivata till $f(x) = \dfrac{\sqrt{x}}{x^2} - \dfrac{3}{x^2}$ som uppfyller $F(1) = 4$. (3p)

   Lösning:

   $$f(x) = x^{-3/2} - 3x^{-2} = \frac{d}{dx}\left[-2x^{-1/2} - 3\frac{x^{-1}}{-1} + C\right]$$

   $$-2+3+C = 4 \qquad C = 3$$

   Svar: $-\dfrac{2}{\sqrt{x}} + \dfrac{3}{x} + 3$

Var god vänd!

(d) Beräkna $\displaystyle\int_0^{\ln 2} e^{-x}(2e^{3x}+4)\,dx$. (3p)

Lösning:

$$2e^{2x}+4e^{-x} = \frac{d}{dx}\left[e^{2x}-4e^{-x}+c\right]$$

$$F(0)=0 \Rightarrow c=3 \Rightarrow F(\ln 2) = 2^2 - \frac{4}{2} + c$$

$$= 5$$

Svar: 5

(e) Lös differentialekvationen $y''+4y = 2x+\sin 3x$. (3p)

Lösning:

$$r^2+4=0 \Leftrightarrow r=\pm 2i \quad y_h = C_1\cos 2x + C_2 \sin 2x$$

$$y_{p_1} = Ax+B \Rightarrow y_{p_1}''+4y_{p_1} = 4Ax+4B = 2x \quad A=\tfrac{1}{2} \quad B=0$$

$$y_{p_2} = A\cos 3x + B\sin 3x \Rightarrow y_{p_2}''+4y_{p_2} = -5A\cos 3x - 5B\sin 3x$$

$$A=0 \quad B=-1/5$$

Svar: $C_1\cos 2x + C_2\sin 2x - 1/5 \sin 3x + x/2$

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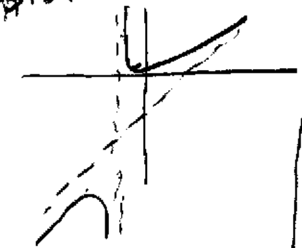
2/ $\frac{(2x+4-3)^2}{x+2} = \underbrace{4(x+2)-12}_{\text{asymptot}} + \frac{9}{x+2}$ $\quad f' = \frac{4(2x+1)(x+2)-(2x+1)^2}{(x+2)^2} = \frac{(2x+1)(2x+7)}{(x+2)^2}$

$-7/2 \quad -2 \quad -1/2$

$+ \; 0 \; - \quad - \; 0 \; +$



$f(-1/2)=0 \quad f(-7/2)=-24$

3/ $(x-2)\frac{d}{dx}\left[-\frac{\cos 2x}{2}\right]$

a/ $= \frac{d}{dx}\left[(x-2)\left(-\frac{\cos 2x}{2}\right)\right]$

$-1\cdot\left(-\frac{\cos 2x}{2}\right) = \frac{d}{dx}\left[-\frac{1}{2}(x-2)\cos 2x + \frac{\sin 2x}{4} + C\right]$

$1+0+C=5 \quad C=4$

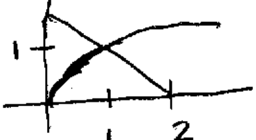
3b/ $\frac{\pi}{2} \quad 2 \quad x$

$f' \quad - \; 0 \; + \; 0 \; -$

4/ $\frac{1}{2x+\sqrt{x}} = \left\{x=t^2\right\} = \frac{1}{2t^2+t}\cdot\frac{dx}{dt}\frac{dt}{dx} = \frac{2t}{t(2t+1)}\frac{dt}{dx} = \frac{2}{2t+1}\frac{dt}{dx}$

$= \frac{d}{dt}\left[\ln|2t+1|\right]\frac{dt}{dx} = \frac{d}{dx}\left[\ln|2t+1|\right] \quad \left[\ln|2t+1|\right]_0^1 = \ln 3$

5/



$x: \; \pi\left(\int_0^1 (\sqrt{x})^2 dx + \int_1^2 (2-x)^2 dx\right) = \pi\left(\left[\frac{x^2}{2}\right]_0^1 + \left[\frac{(x-2)^3}{3}\right]_1^2\right) = \pi\left(\frac{1}{2}+\frac{1}{3}\right)$

$y: \; \pi\int_0^1 \left((2-y)^2-(y^2)^2\right)dy = \pi\left[\frac{(y-2)^3}{3} - \frac{y^5}{5}\right]_0^1 = \pi\left(\frac{8}{3}-\frac{1}{3}-\frac{1}{5}\right)$

6/ $y' + \frac{1}{x}y = \frac{1}{x^3} \quad \int \frac{1}{x}dx = \ln x \quad e^{\ln x} = x$

$(xy)' = \frac{1}{x^2} \quad xy = -\frac{1}{x} + C \quad y = -\frac{1}{x^2} + \frac{C}{x} \quad 1 = -1 + C$

$C = 2$

7i/ t.ex $\frac{1}{\cos^2 x}\frac{1}{\cos^2 x} = \frac{d}{dx}[\tan x]\cdot\frac{1}{\cos^2 x} = \frac{d}{dx}\left[\tan x\cdot\frac{1}{\cos^2 x}\right] - \tan x\frac{d}{dx}\left[\frac{1}{\cos^2 x}\right]$

$= \frac{d}{dx}\left[\frac{\tan x}{\cos^2 x}\right] - \tan x(-2)\cos^{-3}x(-\sin x)$ trig ettan + kickback

ii/ $\frac{x^2}{(x+1)^2} = \frac{(x+1)^2-2(x+1)+1}{(x+1)^2} = 1 - \frac{2}{x+1} + \frac{1}{(x+1)^2} = \frac{d}{dx}\left[\cdots\right]$

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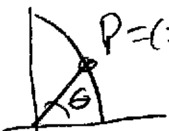

$P=(x, 6-x^2) \quad \tan\theta = \frac{6-x^2}{x} \quad \frac{d\theta}{dt} = 1 \Rightarrow \frac{dx}{dt} = \ldots$

$A = \frac{1}{2}x(6-x^2) \quad \frac{dA}{dt} = \frac{dA}{dx}\frac{dx}{dt}$

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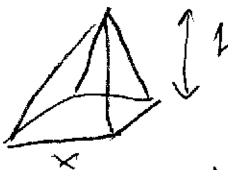

$h \quad V = \frac{x^2 h}{3}$

$x/\sqrt{2} \quad h \quad \sqrt{h^2+\frac{x^2}{2}} \quad x/\sqrt{2}$

$L = 4x + 4\sqrt{h^2+\frac{x^2}{2}} \quad \frac{dL}{dx} = 0$