Anonym kod	MVE525 Matematisk analys 180824	Sidnr 1	Poäng

1. Till nedanstående uppgifter skall korta lösningar redovisas, samt svar anges, på anvisad plats (endast lösningar och svar på detta blad, och på anvisad plats, beaktas).

(a) Bestäm a så att arean under grafen $f(x) = 1 + a/x^3$ mellan $x = 1$ och $x = 2$ är dubbelt så stor som arean mellan $x = 2$ och $x = 3$. (4p)

Lösning:

$$\int_1^2 f(x)dx = 2\int_2^3 f(x)dx \quad \left[x - \frac{a}{2x^2}\right]_1^2 = 2\left[x - \frac{a}{2x^2}\right]_2^3$$

$$1 + \frac{a}{2} - \frac{a}{8} = 2 + \frac{a}{4} - \frac{a}{9} \iff \frac{36 - 9 - 18 + 8}{72} a = 1$$

Svar: $a = \frac{72}{17}$

(b) Bestäm inflexionspunkter till funktionen $f(x) = 5x^3 - 6x^4$. Ange intervall där funktionen är uppåt resp nedåt konkav. (dvs konvex/konkav) (3p)

Lösning:

$$f'(x) = 15x^2 - 24x^3 \quad f''(x) = 30x - 72x^2 = 0$$

$$\iff x = 0 \text{ eller } x = \frac{30}{72} = \frac{5}{12}$$

$$f''(-1) < 0 \quad f''\left(\frac{1}{10}\right) > 0 \quad f''(1) < 0$$

Svar: $x = 0, x = \frac{5}{12}$ CU: $0 < x < \frac{5}{12}$ CD: $x < 0$ resp $x > \frac{5}{12}$

(c) Ange den antiderivata till $f(x) = (\sqrt{x} + 1)^2 + \frac{1}{x^2}$ som uppfyller $F(1) = 2$. (3p)

Lösning:

$$f(x) = x + 2\sqrt{x} + 1 + \frac{1}{x^2} = \frac{d}{dx}\left(\frac{x^2}{2} + \frac{4}{3}x^{3/2} + x - \frac{1}{x} + C\right)$$

$$\frac{1}{2} + \frac{4}{3} + 1 - 1 + C = 2 \quad C = \frac{1}{6}$$

Svar: $F(x) = \frac{x^2}{2} + \frac{4}{3}x\sqrt{x} + x - \frac{1}{x} + \frac{1}{6}$

Var god vänd!

(d) Beräkna $\int_0^{\ln 4} e^{-2x}(e^x + e^{3x})\, dx$. (3p)

Lösning:

$$\int_0^{\ln 4} (e^{-x} + e^{x})\, dx = \left[-e^{-x} + e^{x}\right]_0^{\ln 4}$$

$$= -\frac{1}{4} + 4 = \frac{15}{4}$$

Svar: 15/4

(e) Lös differentialekvationen $y'' + y = x^2 + \cos 2x$. (3p)

Lösning:

$r^2 + 1 = 0 \quad r = \pm i \quad y_h = A\cos x + B\sin x$

$x^2: \quad y_p = C_1 x^2 + C_2 x + C_3 \quad y_p'' = 2C_1$

$2C_1 + C_1 x^2 + C_2 x + C_3 = x^2 \quad C_1 = +1 \quad C_2 = 0$

$C_3 = -2C_1 = -2$

$\cos 2x: \quad y_p = C_4 \cos 2x + C_5 \sin 2x \qquad -3C_4 = 1$

$y_p'' = -4C_4 \cos 2x - 4C_5 \sin 2x \quad -3C_5 = 0$

Svar: $y = A\cos x + B\sin x - \frac{1}{3}\cos 2x + x^2 - 2$

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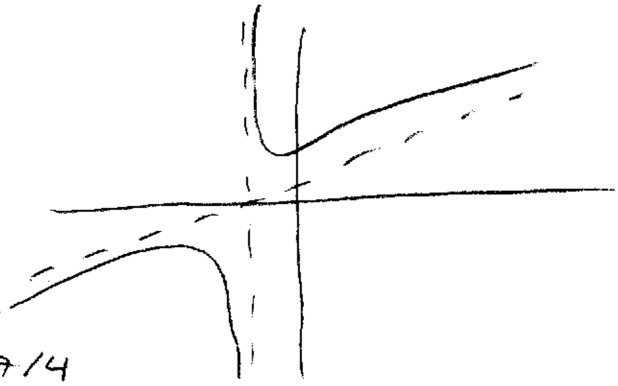
2/ $x \neq -2$ $\lim_{x\to -2^{\pm}} = \pm\infty$ $f(x) - (\frac{x}{2}+1) \to 0, x \to \pm\infty$

$f'(x) = \frac{1}{2} - \frac{1}{(x+2)^2} = \frac{(x+2)^2 - 2}{2(x+2)^2} = 0$ $x = -2 \pm \sqrt{2}$

		$-2-\sqrt{2}$		-2		$-2+\sqrt{2}$	
f'	$+$		$-$		$-$		$+$

$f(-2\pm\sqrt{2}) = \pm\frac{\sqrt{2}}{2} \pm \frac{1}{\sqrt{2}} = \pm\sqrt{2}$



3/ $(x+2)\frac{d}{dx}\left(\frac{e^{2x}}{2} - 2e^x\right)$

$= \frac{d}{dx}\left((x+2)\left(\frac{e^{2x}}{2} - 2e^x\right)\right) - \left(\frac{e^{2x}}{2} - 2e^x\right)$

$= \frac{d}{dx}\left((x+2)\left(\frac{e^{2x}}{2} - 2e^x\right) - \frac{e^{2x}}{4} + 2e^x + C\right)$

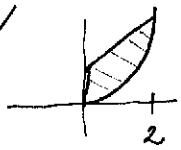
$2\left(\frac{1}{2} - 2\right) - \frac{1}{4} + 2 + C = 3$ $C = 17/4$

4/ $\frac{2}{2-\sqrt{x}} = \left\{\begin{matrix}\sqrt{x} = t \\ x = t^2\end{matrix}\right\} = \frac{2}{2-t}\frac{dx}{dt}\frac{dt}{dx} = \frac{2}{2-t}\cdot 2t\frac{dt}{dx} = \left(-4 + \frac{8}{2-t}\right)\frac{dt}{dx}$

$= \frac{d}{dt}\left(-4t - 8\ln|2-t|\right)\frac{dt}{dx} = \frac{d}{dx}\left(-4t - 8\ln|2-t|\right)$

$\left[-4\sqrt{x} - 8\ln|2-\sqrt{x}|\right]_0^1 = -4 + 8\ln 2$

5/



$x: \pi\int_0^2 \left((x+2)^2 - (x^2)^2\right)dx = \pi\left[\frac{x^3}{3} + 2x^2 + 4x - \frac{x^5}{5}\right]_0^2 = \left(12 + \frac{4}{15}\right)\pi$

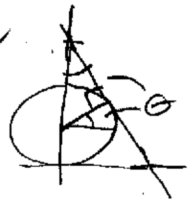
$y: 2\pi\int_0^2 x(x+2-x^2)\,dx = 2\pi\left[\frac{x^3}{3} + x^2 - \frac{x^4}{4}\right]_0^2 = \frac{16\pi}{3}$

6/ $y' + \frac{2}{x}y = 4x$ $e^{\int\frac{2}{x}dx} = x^2$ $(x^2y)' = 4x^3$ $x^2y = x^4 + C$

$y = x^2 + \frac{C}{x^2}$ $2 = 1 + C$ $C = 1$

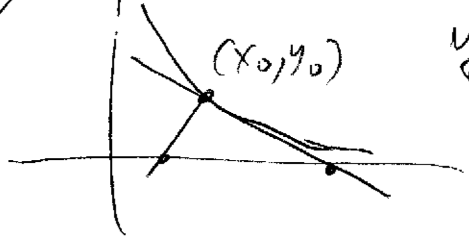
7/ a/ konjugatregel + trigetta + subst $t = \cos x$
b/ subst $t = e^{2x}$

8/



$A = \frac{1}{2}\left(1 + \frac{1}{\sin\theta}\right)^2\tan\theta$ $\frac{dA}{d\theta}$ ---

9/



$y = y_0 - \frac{1}{y_0'}(x - x_0)$ $X = x_0 + y_0y_0'$

$y = y_0 + y_0'(x - x_0)$ $X = x_0 - \frac{y_0}{y_0'}$

$-\frac{y_0}{y_0'} - y_0y_0' = 2$ $y_0' = g(y_0)$ ---