Anonym kod	MVE525 Matematisk analys 180113	Sidnr	Poäng
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1. Till nedanstående uppgifter skall korta lösningar redovisas, samt svar anges, på anvisad plats (endast lösningar och svar på detta blad, och på anvisad plats, beaktas).

(a) Bestäm a så att arean under grafen $f(x) = 1 + a \cdot x^2$ mellan $x = 0$ och $x = 1$ är hälften så stor som arean mellan $x = 1$ och $x = 2$. (4p)

Lösning:

$$2\int_0^1 (1+ax^2)\,dx = \int_1^2 (1+ax^2)\,dx$$

$$2\left[x+\frac{ax^3}{3}\right]_0^1 = \left[x+\frac{ax^3}{3}\right]_1^2$$

$$2\left(1+\frac{a}{3}\right) = 2+\frac{8a}{3}-1-\frac{a}{3} \iff 1 = \frac{5a}{3}$$

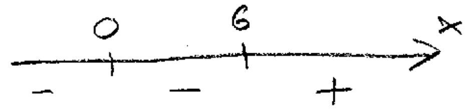
Svar: $a = 3/5$

(b) Bestäm inflexionspunkter till funktionen $f(x) = \dfrac{1}{2x} - \dfrac{1}{x^2}$. Ange intervall där funktionen är uppåt resp nedåt konkav. (dvs konvex/konkav) (3p)

Lösning:

$$f'(x) = -\frac{1}{2x^2}+\frac{2}{x^3} \qquad f''(x) = \frac{1}{x^3}-\frac{6}{x^4} = \frac{x-6}{x^4}$$

f'' :



Svar: $x=6$ CD: $x<0$ resp $0<x<6$ CU: $x>6$

(c) Ange den antiderivata till $f(x) = \dfrac{1}{x^2\sqrt{x}} - \dfrac{2}{x^3}$ som uppfyller $F(1) = 0$. (3p)

Lösning:

$$f(x) = x^{-\frac{5}{2}} - 2x^{-3}$$

$$F(x) = \frac{x^{-\frac{5}{2}+1}}{-\frac{3}{2}} - 2\,\frac{x^{-3+1}}{-2} + C = -\frac{2}{3x\sqrt{x}}+\frac{1}{x^2}+C$$

$$F(1) = -\frac{2}{3}+1+C = 0 \qquad C = -1/3$$

Svar: $F(x) = -\dfrac{2}{3x\sqrt{x}}+\dfrac{1}{x^2}-\dfrac{1}{3}$

Var god vänd!

(d) Beräkna $\int_0^{\pi/2} \cos^2(x)\tan(x)\,dx$. (3p)

Lösning:

$$\cos^2 x \tan x = \cos^2 x \cdot \frac{\sin x}{\cos x} = \cos x \cdot \sin x$$

$$= \left\{ t = \sin x \right\} = \cos x \cdot t \underbrace{\frac{dx}{dt}}_{\frac{1}{\frac{dt}{dx}}} \frac{dt}{dx} = \cos x \cdot t \cdot \frac{1}{\cos x} \cdot \frac{dt}{dx}$$

$$= t \frac{dt}{dx} = \frac{d}{dt}\left(\frac{1}{2}t^2\right)\frac{dt}{dx} = \frac{d}{dx}\left(\frac{1}{2}t^2\right)$$

(alt dubbla vinkel $\cos x \sin x = \frac{1}{2}\sin 2x$)

$$\left[\frac{1}{2}t^2\right]_0^1 = \frac{1}{2}$$

Svar: ....

(e) Lös differentialekvationen $y'' + 4y' = 3 + e^{2x}$. (3p)

Lösning:

$$r^2 + 4r = 0 \qquad r = 0 \vee r = -4$$

$$y_h = c_1 e^{0x} + c_2 e^{-4x} = c_1 + c_2 e^{-4x}$$

$$y_p = Ax + Be^{2x}$$

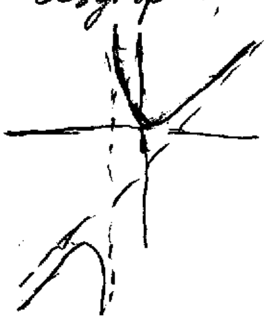
$$y_p' = A + 2Be^{2x}$$

$$y_p'' = 4Be^{2x}$$

$$y_p'' + 4y_p' = \underbrace{4A}_{3} + \underbrace{8B}_{1} e^{2x}$$

Svar: $y = c_1 + c_2 e^{-4x} + \frac{3}{4}x + \frac{1}{8}e^{2x}$

2 $f' = \frac{8x(2x+1) - 4x^2 \cdot 2}{(2x+1)^2} = \frac{x(8x+8)}{(2x+1)^2}$ $\quad f(x) = \underbrace{2x-1}_{\text{asymptot}} + \frac{1}{2x+1}$



$-1 \quad -1/2 \quad 0 \quad x$

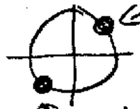
$+ \quad - \quad - \quad +$

x	y	
-1	-4	← lok max
0	0	← lok min

$\lim_{x \to -\frac{1}{2}\pm} f(x) = \pm\infty$

3 $f(x) = -\frac{1}{2}\cos 2x + \frac{1}{2}\sin 2x + C \quad f(0) = -\frac{1}{2} + C$

$C = 9/2$



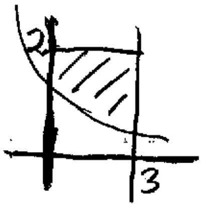
← max ← $x = \frac{\pi}{8} + n\frac{\pi}{2},\ n = \pm 0, 2, 4, 6$

↑ min — $x = \frac{\pi}{8} + n\frac{\pi}{2},\ n = \pm 1, 3, 5, 7$

4 $x D\left(\frac{e^{-2x}}{-2} + 2\frac{e^{-x}}{-1}\right) = D\left[x\left(\frac{e^{-2x}}{-2} + 2\frac{e^{-x}}{-1}\right)\right] - \underbrace{\overbrace{(Dx)}^{=1}\left(\frac{e^{-2x}}{-2} + 2\frac{e^{-x}}{-1}\right)}_{D\left(\frac{e^{-2x}}{4} + 2e^{-x}\right)}$

$\left[x\left(\frac{e^{-2x}}{-2} + 2\frac{e^{-x}}{-1}\right) - \left(\frac{e^{-2x}}{4} + 2e^{-x}\right)\right]_0^\infty = 0 + \frac{1}{4} + 2 = 9/4$

5



$x: \int_0^3 \pi\left(2^2 - \frac{1}{(x+1)^2}\right)dx = 12\pi + \pi\left[\frac{1}{x+1}\right]_0^3 = \frac{45\pi}{4}$

$y: \int_0^3 2\pi x\left(2 - \frac{1}{x+1}\right)dx = 2\pi\left[x^2 - x + \ln|x+1|\right]_0^3 = 4\pi(3 + \ln 2)$

6 $y' - \frac{1}{x}y = x^3\sqrt{x} \quad \text{IF}: e^{-\int \frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

$\frac{1}{x}y' - \frac{1}{x^2}y = x^2\sqrt{x} \quad D\left(\frac{1}{x}y\right) = x^2\sqrt{x} \quad \frac{1}{x}y = \frac{2}{7}x^3\sqrt{x} + C$

$\frac{1}{1}\cdot 1 = \frac{2}{7} + C \quad C = \frac{5}{7} \quad y = \frac{2x^4\sqrt{x} + 5x}{7}$

7 Tex dubbla vinkel resp konjugatregel

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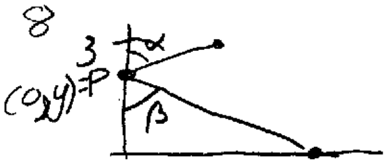

Tex vinkel $= \pi - \alpha - \beta = \pi - \arctan(\ ) - \arctan(\ )$

$\frac{d}{dy} = 0$ -----

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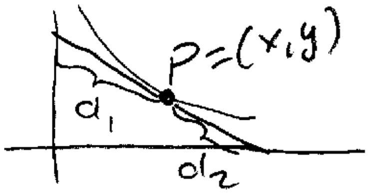

$d_1 = d_2$ ställ upp samband x, y, y' ---