Anonym kod	MVE525 Matematisk analys 190119	Sidnr	Poäng
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1. Till nedanstående uppgifter skall korta lösningar redovisas, samt svar anges, på anvisad plats (endast lösningar och svar på detta blad, och på anvisad plats, beaktas).

(a) Bestäm arean av området som begränsas av $y = 3 - x^2$ och $y = x + 1$. (4p)

Lösning:

$3 - x^2 = x + 1 \qquad x = \dfrac{-1 \pm \sqrt{1+8}}{2}$

$$\int_{-2}^{1} (x^2 + x - 2)\,dx = \left[\frac{x^3}{3} + \frac{x^2}{2} - 2x\right]_{-2}^{1} = -\frac{9}{2}$$

Svar: $\frac{9}{2}$

(b) Bestäm inflexionspunkter till funktionen $f(x) = 6x^2 - x^4$. Ange intervall där funktio~~nen är uppåt resp nedåt konkav.~~ (dvs konvex/konkav) (3p)

Lösning:

$f''(x) = 12 - 12x^2 = 0 \iff x = \pm 1$

$-1 \quad 1 \quad x$

$- \quad 0 \quad + \quad 0 \quad -$

CD CU CD

$-2 \to -$

$0 \to +$

$2 \to -$

Svar: $x = \pm 1$ konkav: $x < -1$ resp $x > 1$ konvex: $-1 < x < 1$

(c) Ange den antiderivata till $f(x) = \dfrac{1+2x}{x^3}$ som uppfyller $F(1) = 2$. (3p)

Lösning:

$f(x) = x^{-3} + 2x^{-2}$

$F(x) = \dfrac{x^{-2}}{-2} + 2\dfrac{x^{-1}}{-1} + C \qquad F(1) = -\dfrac{1}{2} - 2 + C$

Svar: $F(x) = -\dfrac{1}{2x^2} - \dfrac{2}{x} + \dfrac{9}{2}$

Var god vänd!

(d) Beräkna $\int_0^{\ln 5} e^{-3x}(e^x + 2e^{4x})\,dx$. (3p)

Lösning:

$$e^{-2x} + 2e^x = \frac{d}{dx}\left(\frac{e^{-2x}}{-2} + 2e^x + C\right)$$

$$\frac{e^{-2\ln 5}}{-2} + 2e^{\ln 5} - \left(-\frac{1}{2} + 2\right) = 8 + \frac{1}{2} - \frac{1}{50}$$

Svar: $8 + \frac{48}{100}$

(e) Lös differentialekvationen $y'' + 4y = 3x^2 + 2x + 1$. (3p)

Lösning:

$$r^2 + 4 = 0 \quad r = \pm 2i$$

$$y_h = C_1 \cos 2x + C_2 \sin 2x$$

$$y_p = ax^2 + bx + c$$

$$y_p'' + 4y_p = 2a + 4ax^2 + 4bx + 4c$$

$$\left.\begin{aligned} 2a + 4c &= 1 \\ 4b &= 2 \\ 4a &= 3 \end{aligned}\right\} \quad a = \frac{3}{4} \quad b = \frac{1}{2} \quad c = -\frac{1}{8}$$

Svar: $y = \frac{3}{4}x^2 + \frac{1}{2}x - \frac{1}{8} + C_1 \cos 2x + C_2 \sin 2x$

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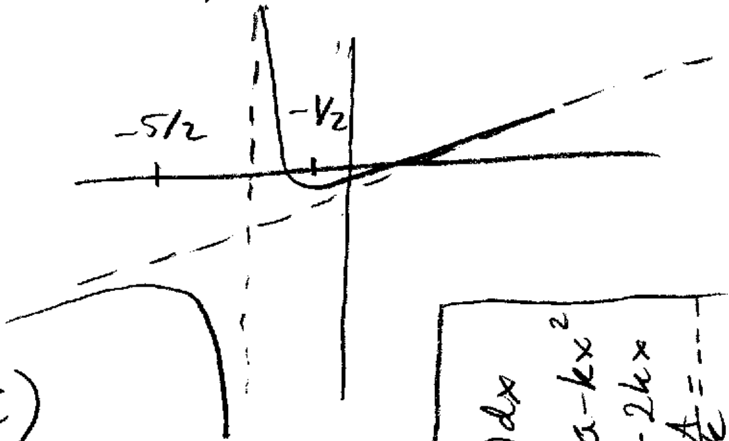
2 $x \neq -3/2$ $\quad \lim_{x\to -3/2^{\pm}} f(x) = \pm\infty$ $\quad f(x) - (-1+\frac{x}{2}) \to 0,\ x \to \pm\infty$

$f'(x) = \frac{1}{2} - \frac{2}{(2x+3)^2}$ $\quad 2x+3 = \pm 2$ $\quad x = \begin{cases} -5/2 \\ -1/2 \end{cases}$

$\xrightarrow[\;+\;0\;-\;|\;-\;0\;+\;]{-5/2 \quad -3/2 \quad -1/2} x$

$f(-5/2) = \quad , f(-1/2) =$



3 a $f(x) = \frac{d}{dx}\left((2x-3)(e^x + e^{-2x})\right)$

$-2\cdot(e^x + e^{-2x}) =$

$= \frac{d}{dx}\left((2x-3)(e^x+e^{-2x}) - 2e^x + e^{-2x} + C\right)$

$f(0) = -6-2+1+C \quad C = 11$

b $x = \frac{3}{2},\ e^x = 2e^{-2x},\ e^{3x} = 2,\ x = \frac{\ln 2}{3}$ $\qquad 0 \to +$, $1 \to -$, $2 \to +$

$\xrightarrow[\;+\;0\;-\;0\;+\;]{(\ln 2)/3 \qquad 3/2}$

max $\qquad$ min

4 $\frac{\sqrt{x}}{2+\sqrt{x}} = \left\{\begin{array}{l} 2+\sqrt{x} = t \\ x = (t-2)^2 \end{array}\right\} = \frac{dF}{dx} = \frac{dF}{dt}\cdot\underbrace{\frac{dt}{dx}}_{\frac{1}{2\sqrt{x}}}$

$\frac{dF}{dt} = \frac{2x}{t} = \frac{2(t-2)^2}{t} = 2\left(t-4+\frac{4}{t}\right) \Rightarrow F(t) = t^2 - 8t + 8\ln|t| + C$

$x=0 \Rightarrow t=2$
$x=1 \Rightarrow t=3$ $\qquad \left[t^2 - 8t + 8\ln|t|\right]_2^3 = -3 + 8\ln\frac{3}{2}$

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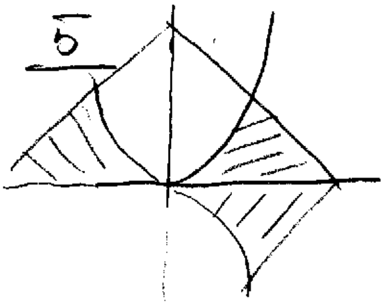


$x:\ \pi\int_0^1 (x^3)^2\,dx + \pi\int_1^2 (2-x)^2\,dx = \frac{\pi}{7} + \frac{\pi}{3}$

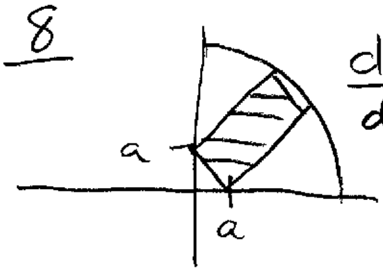
$y:\ 2\pi\int_0^1 x^3\cdot x\,dx + 2\pi\int_1^2 (2-x)x\,dx = \frac{2\pi}{5} + \frac{4\pi}{3}$

6 $y' + \frac{1}{x}y = \sin 2x$ $\quad e^{\int \frac{1}{x}dx} = e^{\ln x} = x$ $\quad (xy)' = x\sin 2x = \frac{d}{dx}\left(x\cdot\frac{-\cos 2x}{2}\right)$

$+\frac{\cos 2x}{2} = \frac{d}{dx}\left(-\frac{x}{2}\cos 2x + \frac{\sin 2x}{4} + C\right)$ $\quad y = -\frac{\cos 2x}{2} + \frac{\sin 2x}{4x} + \frac{C}{x}$

$1/2 + 2C/\pi = 1 \quad C = \pi/4$

7a $\frac{1}{2\cos^2 x} + \underbrace{\frac{1}{\cos x}}_{\frac{1+\tan^2\frac{x}{2}}{1-\tan^2\frac{x}{2}}} + \frac{1}{2}$ $\qquad$ **7b** $\frac{x^{11}}{(x^6+1)^2}$ $\qquad$ **8**



$\frac{dA}{da} = \ldots$

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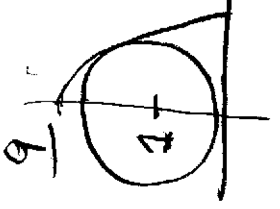

$A = \int_0^{\sqrt{a/k}} (a-kx^2)\,dx$

$y = 1+\sqrt{1-x^2} = a - kx^2$

$y' = -\frac{x}{\sqrt{1-x^2}} = -2kx$

$a = a(k) \quad \frac{dA}{dk} = \ldots$