Anonym kod	MVE525/LMA515	190823	Sida	Poäng
			1	

1. Till nedanstående uppgifter skall korta lösningar redovisas, samt svar anges, på anvisad plats (endast lösningar och svar på detta blad, och på anvisad plats, beaktas).

(a) Bestäm arean av området som begränsas av $y = 2x - 2$ och $y = x^2 - x$. (4p)

Lösning:

$$x^2 - x = 2x - 2 \qquad x = \frac{3}{2} \pm \sqrt{\frac{9}{4} - 2} = \begin{cases} 2 \\ 1 \end{cases}$$

$$\int_{1}^{2} -2 + 3x - x^2 = \left[-2x + \frac{3x^2}{2} - \frac{x^3}{3}\right]_1^2 = -2 + \frac{3}{2}\cdot 3 - \frac{1}{3}\cdot 7$$

Svar: $-2 + \frac{9}{2} - \frac{7}{3}$

(b) Bestäm inflexionspunkter till funktionen $f(x) = x^3 + x^4$. Ange intervall där funktionen är uppåt resp nedåt konkav. (dvs konvex/konkav) (3p)

Lösning:

$$f' = 3x^2 + 4x^3 \qquad f'' = 6x + 12x^2 = 6x(1 + 2x)$$

$$x = 0 \qquad x = -\frac{1}{2}$$

		$-1/2$		0		x
f''	$+$		$-$		$+$	
	CU		CD		CU	

Svar: $x = 0, x = -1/2$ CU: $x < -1/2$ resp $x > 0$ CD: $-1/2 < x < 0$

(c) Ange den antiderivata till $f(x) = \dfrac{4 + x^2}{x^5}$ som uppfyller $F(1) = 1$. (3p)

Lösning:

$$4x^{-5} + x^{-3} = \frac{d}{dx}\Big(\underbrace{4\frac{x^{-4}}{-4} + \frac{x^{-2}}{-2} + C}_{F(x)}\Big)$$

$$F(1) = -1 - \frac{1}{2} + C = 1 \qquad C = 5/2$$

Svar: $F(x) = -\frac{1}{x^4} - \frac{1}{2x^2} + \frac{5}{2}$

Var god vänd!

(d) Beräkna $\int_0^{\ln 2} e^{2x}(e^{3x} + e^{-2x})\, dx$. (3p)

Lösning:

$$e^{2x+3x} + e^{2x-2x} = e^{5x} + 1 = \frac{d}{dx}\left(\frac{e^{5x}}{5} + x\right)$$

$$\left[\frac{e^{5x}}{5} + x\right]_0^{\ln 2} = \frac{2^5}{5} + \ln 2 - \frac{1}{5}$$

Svar: $31/5 + \ln 2$

(e) Lös differentialekvationen $y'' + 3y' - 4y = 5e^{2x}$. (3p)

Lösning:

$$r^2 + 3r - 4 = 0 \qquad r = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + 4} = \frac{-3 \pm 5}{2}$$

$$y_h = C_1 e^{-4x} + C_2 e^{x}$$

$$y_p = Ae^{2x} \qquad y_p' = 2Ae^{2x} \qquad y_p'' = 4Ae^{2x}$$

$$(4A + 3 \cdot 2A - 4 \cdot A)e^{2x} = 6Ae^{2x} \qquad A = \frac{5}{6}$$

Svar: $y = C_1 e^{-4x} + C_2 e^{x} + \frac{5}{6} e^{2x}$

SVAR MVE525 23/8 – 2019

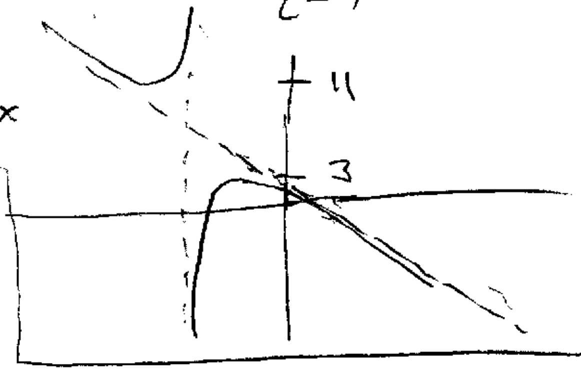
2 $\quad x \neq -3 \quad \lim_{x \to \pm -3} f(x) = \mp \infty \qquad f(x) - (1-2x) \to 0,\ x \to \pm\infty$

$f'(x) = -2 + \frac{2}{(x+3)^2} \qquad f'(x) = 0 \Leftrightarrow x+3 = \pm 1 \Leftrightarrow x = \begin{cases} -2 \\ -4 \end{cases}$

$$\begin{array}{lcccccc} & & -4 & & -3 & & -2 & & x \\ f'(x) & - & & + & & + & & - \end{array}$$

$f(-4) = 11$ lok min

$f(-2) = 3$ lok max



3a $f(x) = \int (2x+3)(2-e^{2x})\,dx =$

$= \left(2x - \frac{e^{2x}}{2}\right)(2x+3) - \int \left(2x - \frac{e^{2x}}{2}\right) 2\,dx$

$= \left(2x - \frac{e^{2x}}{2}\right)(2x+3) - 2x^2 + \frac{e^{2x}}{2} + C \qquad f(0) = -\frac{3}{2} + \frac{1}{2} + C = 1$

$C = 2$

3b $\quad 2x+3 = 0 \quad x = -3/2$

$2 = e^{2x} \quad x = \frac{\ln 2}{2}$

$$\begin{array}{ccccc} & -3/2 & & \frac{\ln 2}{2} & \\ - & \uparrow & + & \uparrow & - \\ & \text{min} & & \text{max} & \end{array}$$

test

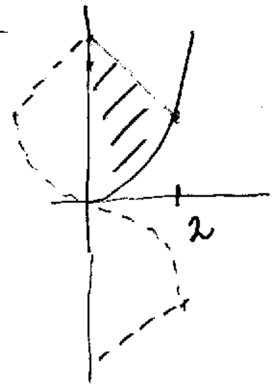
$-5 \Rightarrow -$

$0 \Rightarrow +$

$1 \Rightarrow -$

4 $\quad 1+x = t \quad \int_0^1 (t-1)^2 \sqrt{t}\,dt = \left[\frac{t^{7/2}}{7/2} - \frac{2t^{5/2}}{5/2} + \frac{t^{3/2}}{3/2}\right]_0^1 = \frac{2}{7} - \frac{4}{5} + \frac{2}{3}$

5



$x:\ \pi \int_0^2 \left((6-x)^2 - (x^2)^2\right) dx = \pi \left[36x - 6x^2 + \frac{x^3}{3} - \frac{x^5}{5}\right]_0^2$

$y:\ 2\pi \int_0^2 \left((6-x) - x^2\right) x\,dx = 2\pi \left[3x^2 - \frac{x^3}{3} - \frac{x^4}{4}\right]_0^2$

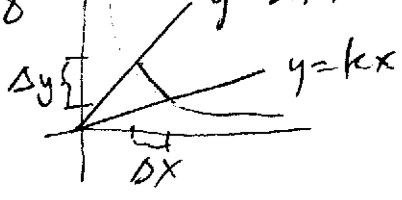
6 $\quad \frac{dy}{y} = -3\sqrt{x}\,dx \qquad \ln|y| = -2x\sqrt{x} + C \qquad y = \underbrace{\pm e^{C}}_{K} e^{-2x\sqrt{x}}$

$1 = K e^{-2} \quad K = e^2 \quad y = e^{2 - 2x\sqrt{x}}$

7a $\quad x^2 = t$ ....

7b $\quad \sin x = t$ ...

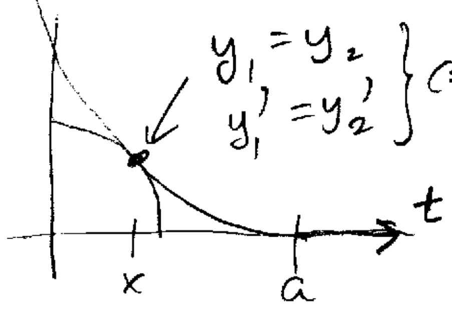
8



$L^2 = \Delta x^2 + \Delta y^2 = L^2(k)$

$\frac{dL^2}{dk} = 0$ alt symmetri + $L^2 \geq L^2_{min}$

9



$\left.\begin{array}{l} y_1 = y_2 \\ y_1' = y_2' \end{array}\right\} \Leftrightarrow \begin{array}{l} a = a(x) \\ b = b(x) \end{array} \qquad A(a,b) = \int_0^a b(t-a)^2\,dt = A(x)$

$\frac{dA}{dx} = 0$