Anonym kod	MVE535 Matematisk analys, del 1 180315	Sidnr 1	Poäng

1. Till nedanstående uppgifter skall lösningar redovisas, samt svar anges, på anvisad plats (endast lösningar och svar på detta blad, och på anvisad plats, beaktas).

(a) Bestäm med hjälp av derivatans definition $f'(x)$ då $f(x) = \dfrac{2x}{x+1}$. (2p)

Lösning:

$$f'(x) = \lim_{x \to 0} \left(\frac{2(x+h)}{x+h+1} - \frac{2x}{x+1} \right) \frac{1}{h}$$

$$= \lim_{x \to 0} \frac{2x^2 + 2x + 2xh + 2h - 2x^2 - 2xh - 2x}{h(x+h+1)(x+1)}$$

$$= \lim_{x \to 0} \frac{2}{(x+h+1)(x+1)} = \frac{2}{(x+1)^2}$$

Svar: ....

(b) Bestäm lokala max/min till funktionen $f(x) = \dfrac{1}{x} + 3\ln x$. (3p)

Lösning:

$$f'(x) = -\frac{1}{x^2} + \frac{3}{x} = \frac{-1+3x}{x^2}$$

$$f'(x) = 0 \iff x = 1/3 \qquad f'(1) > 0$$

$$f'(1/4) < 0$$

$1/3$, x

$-$ $+$

Svar: lokalt min i $x = 1/3$

(c) Lös ekvationen $|x^2 - 4| + 4 = 3x$. (3p)

Lösning:

$x \geq 2$ eller $x \leq -2$: $x^2 = 3x$ $x = 0$

eller $x = 3$ ej i intervall

$-2 < x < 2$: $4 - x^2 + 4 = 3x$

$$x = \frac{-3 \pm \sqrt{9 + 4 \cdot 8}}{2} = \frac{-3 \pm \sqrt{41}}{2}$$

Svar: $x = 3$ eller $x = (-3 + \sqrt{41})/2$

Var god vänd!

(d) Ange den antiderivata till $f(x) = \dfrac{1}{(x+1)^2} - \dfrac{3}{x^4}$ som uppfyller $F(1) = 0$. (2p)

Lösning:

$$f(x) = \frac{d}{dx}\Big[\underbrace{-\frac{1}{x+1} - 3\cdot\frac{x^{-3}}{-3} + C}_{F(x)}\Big]$$

$$0 = -\frac{1}{2} + 1 + C \iff C = -\frac{1}{2}$$

Svar: $F(x) = -1/(x+1) + 1/x^3 - 1/2$

(e) Bestäm inversen till funktionen $f(x) = \left(\dfrac{1}{\sqrt{x}} + 2\right) / \left(\dfrac{3}{\sqrt{x}} + 4\right)$. (3p)

Lösning:

$$y = f(x) \iff x = f^{-1}(y)$$

$$\frac{1 + 2\sqrt{x}}{3 + 4\sqrt{x}} = y \iff 1 + 2\sqrt{x} = 3y + 4y\sqrt{x}$$

$$\iff \sqrt{x} = \frac{1 - 3y}{4y - 2}$$

Svar: $f^{-1}(x) = \left(\dfrac{1-3x}{4x-2}\right)^2$

(f) Bestäm ekvationen för tangenten till f's graf i punkten där $x = 3$ för funktionen $f(x) = (x-2)\sqrt{x+1}$. Ange också tangentens skärningspunkt med x-axeln. (3p)

Lösning:

$$f'(x) = \sqrt{x+1} + \frac{x-2}{2\sqrt{x+1}} \qquad f'(3) = 2 + \frac{1}{4}$$

$$y = 2 + \frac{9}{4}\cdot(x-3)$$

$$y = 0 \iff -2 = \frac{9}{4}(x-3) \iff x = 3 - \frac{8}{9}$$

Svar: $y = 2 + \dfrac{9}{4}(x-3) \qquad x = 3 - \dfrac{8}{9} = \dfrac{19}{9}$

$$= \frac{9x}{4} - \frac{19}{4}$$

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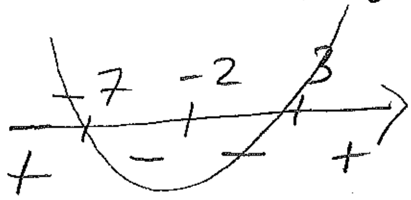
2/ $f' = \dfrac{2(x-3)(2x+4)-(x-3)^2\cdot 2}{(2x+4)^2}$, $x \neq -2$

$= \dfrac{2(x-3)((2x+4)-x+3)}{(2x+4)^2} = \dfrac{2(x-3)(x+7)}{(2x+4)^2}$



$f(-7) = -10$ lok max

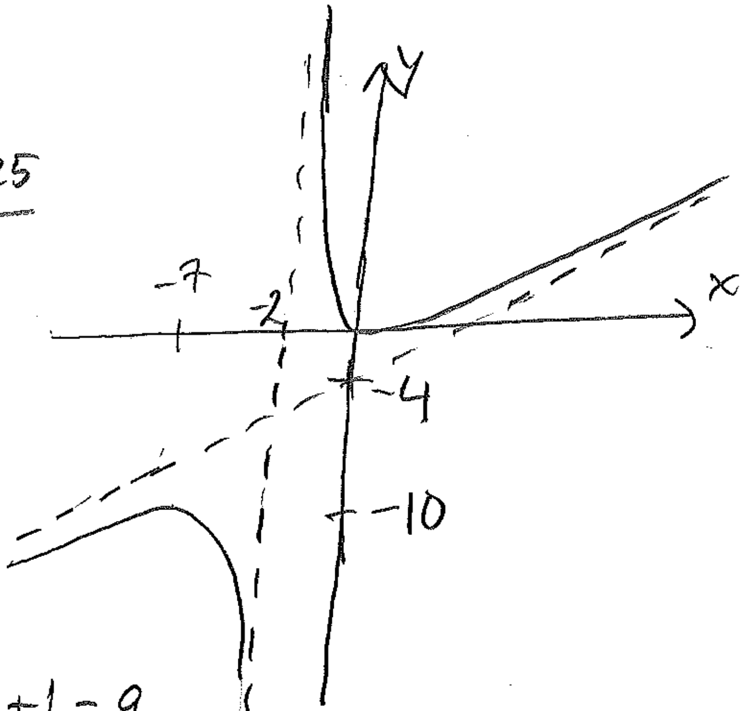
$f(3) = 0$ lok min

$f(x) = \dfrac{(x+2)^2 - 10(x+2) + 25}{2(x+2)}$

$= \underbrace{\dfrac{x+2}{2} - 5} + \dfrac{25}{2(x+2)}$

sned asymptot

$\lim_{x \to -2^{\pm}} = \pm\infty$



3/ $(x+3)^2 + 4(y+\tfrac{1}{2})^2 = -1+9+1 = 9$

a/ $\dfrac{(x+3)^2}{3^2} + \dfrac{(y+\frac{1}{2})^2}{(\frac{3}{2})^2} = 1$

centr: $(-3, -1/2)$

störst: 3

minst: 3/2

b/ $x=-2 \Rightarrow 4y^2 + 4y = 7$

$y = -\frac{1}{2} \pm \sqrt{2}$

$2x + 6 + 8yy' + 4y' = 0$

$y' = -\dfrac{2x+6}{8y+4} = -\dfrac{1}{\pm\sqrt{2}}$

4/ $\lim_{x\to 0} \overset{\frac{0}{0}}{\underset{H}{=}} \lim_{x\to 0} \dfrac{2\cos 2x - 2\cos x + 15x^2}{3x^2} \overset{\frac{0}{0}}{\underset{H}{=}} \lim_{x\to 0} \dfrac{-4\sin 2x + 2\sin x + 30x}{6x}$

$\overset{\frac{0}{0}}{\underset{H}{=}} \lim_{x\to 0} \dfrac{-8\cos 2x + 2\cos x + 30}{6} = \dfrac{24}{6} = 4$

5/ $y' = (2c_1x^2 + 2c_2x + 2c_1x + c_2)e^{2x}$ $\quad y'' = (4c_1x^2 + 4c_2x + 4c_1x + 2c_2 + 4c_1x + 2c_2 + 2c_1)e^{x}$

$y'' - 2y' = (4c_1x + 2c_2 + 2c_1)e^{2x} = (3x+4)e^{2x}$ $\quad c_1 = \frac{3}{4}$ $\quad c_2 = \frac{5}{4}$

6/ $x^2 - 8x + 16 = 9x^2 + 24x + 16 \Leftrightarrow 8x^2 + 32x = 0 \Leftrightarrow 8x(x+4) = 0$

test $x = 0$ ej lösn $\quad \underline{x = -4}$ lösn

7/a/ $f(x) = \frac{3}{-2}e^{-2x} + \frac{2}{4}e^{-4x} + C$ $\quad 5 = -\frac{3}{2} + \frac{1}{2} + C \Leftrightarrow C = 6$

b/ $3 = 2e^{-2x}$ $\quad x = -\frac{1}{2}\ln\frac{3}{2}$ $\quad f'(-\ln 10) = -1700$, $f'(0) = 1$ $\quad - \; +$ lokmin

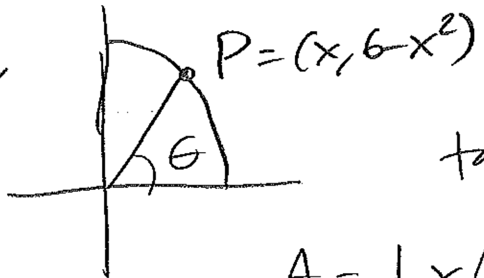
8i/ T.ex. $\frac{1}{\cos^4 x} = \frac{d}{dx}[\tan x]\frac{1}{\cos^2 x} = \frac{d}{dx}\left[\tan x \frac{1}{\cos^2 x}\right]$

$- \tan x \frac{d}{dx}\left[\frac{1}{\cos^2 x}\right]$ följt av trig etta + kickback

8ii/ $\frac{x^2}{(x+1)^2} = \frac{(x+1)^2 - 2(x+1) + 1}{(x+1)^2} = 1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}$

$= \frac{d}{dx}[\quad]$

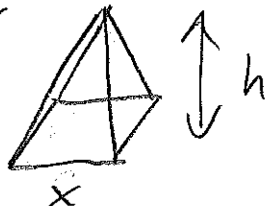
9/



$\tan\theta = \frac{6-x^2}{x}$ $\quad \frac{d\theta}{dt} = 1 \Rightarrow \frac{dx}{dt}$

$A = \frac{1}{2}x(6-x^2)$ $\quad \frac{dA}{dt} = \frac{dA}{dx}\frac{dx}{dt}$

10/



$V = \frac{x^2 h}{3}$ $\quad L = 4x + 4\cdot\sqrt{\ldots\ldots}$

$\frac{dL}{dx} = 0$