MVE455 Dugga Lösningsförslag 170205

1) a) $\int \frac{1}{4x^2+1}\,dx = \int \frac{1}{(2x)^2+1}\,dx = [t = 2x] = \frac{1}{2}\int \frac{1}{t^2+1}\,dt = \frac{1}{2}\arctan(2x) + C$

b) $\int_0^{\pi/4} \tan x\,dx = \int \frac{1}{\cos x}\sin x\,dx = [t = \cos x] = -\int_1^{1/\sqrt{2}} \frac{1}{t}\,dt = \int_{1/\sqrt{2}}^{1} \frac{1}{t}\,dt = [\ln|t|]_{1/\sqrt{2}}^{1} =$

$= \ln 1 - \ln 2^{-1/2} = \frac{1}{2}\ln 2$

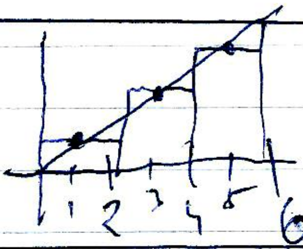
c) $\int x e^{-x}\,dx = [PI] = x(-e^{-x}) - \int 1\cdot(-e^{-x})\,dx = -xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + C = -(x+1)e^{-x} + C$

2) $\frac{3}{x^2-5x+4} = \frac{3}{(x-1)(x-4)} = [PBU] = \frac{A}{x-1} + \frac{B}{x-4} = [HP] = \frac{-1}{x-1} + \frac{1}{x-4} \Rightarrow$

$\int \frac{3}{x^2-5x+4}\,dx = \int -\frac{1}{x-1} + \frac{1}{x-4}\,dx = -\ln|x-1| + \ln|x-4| + C$

3) $\frac{\sin(x^3)}{(x^2+x^4)^3} = f(x)$ uppfyller $f(-x) = \frac{\sin((-x)^3)}{((-x)^2+(-x)^4)^3} = \frac{\sin(-x^3)}{(x^2+x^4)^3} = -f(x)$

så f är udda och på symmetriska intervallet $[-2, 2]$ gäller $\int_{-2}^{2} f(x)\,dx = 0$

4) 

$\int_0^6 x\,dx \approx M_3 = 2f(1) + 2f(3) + 2f(5) = (f(x) = x)^2$

$= 2\cdot 1 + 2\cdot 3 + 2\cdot 5 = 18$

5) (Utgick) $y' = ky \Leftrightarrow y' + (-k)y = 0$ Linjär IF: $e^{\int -k\,dx} = e^{-kx}$

$\therefore \frac{d}{dx}(e^{-kx}y) = e^{-kx}\cdot 0 = 0 \Rightarrow e^{-kx}y = \int \frac{d}{dx}(e^{-kx}y)\,dx = \int 0\,dx = C,\ C \in \mathbb{R}$

$\therefore y = Ce^{kx}$

6) $I = \int e^t \sin(2t)\,dt = [PI] = e^t\sin(2t) - \int e^t\cos(2t)\,2\,dt =$

$= e^t\sin(2t) - 2\int e^t\cos(2t)\,dt = [PI] = e^t\sin(2t) - 2\Big(e^t\cos(2t)$

$- \int e^t(-\sin(2t))\,2\,dt\Big) = e^t(\sin(2t) - 2\cos(2t)) - 4I$

$\therefore$ (bootstrapping) $5I = e^t(\sin(2t) - 2\cos(2t)) + C$

$\Rightarrow I = \frac{1}{5}e^t(\sin(2t) - 2\cos(2t)) + C$