Problem 6.7. Let G the Green's function in (3.18) of Sect. 3.4 and let $\{\lambda_j\}_{j=1}^\infty$ and $\{\varphi_j\}_{j=1}^\infty$ be the eigenvalues and normalized eigenfunctions of (6.5) as in Theorem 6.4. Show that

$$G(x,y) = \sum_{j=1}^\infty \lambda_j^{-1}\varphi_j(x)\varphi_j(y).$$