Computer Exercise TMA026 - 2 Bonus Points

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The purpose of this computer exercise is to get acquainted with the FEniCS software by performing a convergence study for an elliptic problem. Consider the following boundary value problem

$$\begin{cases} -\nabla \cdot (a\nabla u) + bu = f & \text{in } \Omega, \\ u = u_D & \text{on } \Gamma_D, \\ -a\frac{\partial u}{\partial n} = u_N & \text{on } \Gamma_N, \\ -a\frac{\partial u}{\partial n} = k(u - u_R) & \text{on } \Gamma_R, \end{cases} \tag{1}$$

for some suitable non-constant functions a, b, f, k and some non-zero functions u_D, u_N, u_R.

Task 1 Create an "exact solution" in the following way. Choose a rather simple domain Ω (rectangle, circle, square) and some simple coefficient functions a, b, k. Then choose a nontrivial function u and compute the corresponding data functions f, u_D, u_N, u_R. That is $f = -\nabla \cdot (a\nabla u) + bu$ and so on.

Task 2 After reading the tutorial available online about `elliptic problems` and `multiple boundary conditions`, download the `FEniCS software` and the `demos` and understand how simpler versions of the problem (1) could be solved using FEniCS, paying particular attention to the difference between single and multiple Neumann/Robin boundary conditions and to how the convergence/error analysis is performed.

Task 3 Compute the finite element solution u_h for the problem (1), for a sequence of meshes, and create a table of the errors $\|u - u_h\|$ and $|u - u_h|_1$ as functions of the mesh-size h. Determine the order of convergence.
This kind of convergence study is not only an illustration of the theory but more importantly it is used as a test of the computer program: a minor programming error can result in a non-optimal convergence rate.

Task 4 - Optional Use finite elements of higher order p and make a convergence analysis as in the previous point.

Task 5 - Optional In the solution of the resulting linear system use different `solvers` and make an heuristic analysis of the outcomes.

Notice that the FEniCS book is freely available `online`. The study of the first chapter, "A FEniCS tutorial", pages $1 - 75$, is strongly recommended.