

Problem 5.19 Show that the *edge midpoint quadrature*

$$q_K(f) = \frac{|K|}{3} \sum_{j=1}^3 f(m_j)$$

is exact for $f \in \Pi_2$. Here the m_j denote the midpoints of the edges of triangle K . Show also that this is not true for the barycentric and the nodal quadratures.

Hint: it is sufficient to prove this for the unit size reference triangle \hat{K} with corners at $(0, 0)$, $(1, 0)$, $(0, 1)$.

This quadrature is useful for computing the mass matrix:

$$M_{ij} = (\Phi_i, \Phi_j) = \sum_K (\Phi_i, \Phi_j)_K.$$