## Stencil N3   Kedjeregeln

Välj av olika typer av problem och lös gärna 25% av dem.

**3124.** $u = e^{x-2y}$, где $x = \sin t$, $y = t^3$, $\dfrac{du}{dt} = ?$

**3125.** $u = z^2 + y^2 + zy$, $z = \sin t$, $y = e^t$; $\dfrac{du}{dt} = ?$

**3126.** $z = \arcsin(x - y)$, $x = 3t$, $y = 4t^3$; $\dfrac{dz}{dt} = ?$

**3127.** $z = x^2 y - y^2 x$, где $x = u \cos v$, $y = u \sin v$; $\dfrac{\partial z}{\partial u} = ?$ $\dfrac{\partial z}{\partial v} = ?$

**3128.** $z = x^2 \ln y$, $x = \dfrac{u}{v}$, $y = 3u - 2v$; $\dfrac{\partial z}{\partial u} = ?$ $\dfrac{\partial z}{\partial v} = ?$

**3129.** $u = \ln(e^x + e^y)$; $\dfrac{\partial u}{\partial x} = ?$ Finn $\dfrac{du}{dx}$, om $y = x^3$.

**3130.** $z = \arctan(xy)$; Finn $\dfrac{dz}{dx}$, om $y = e^x$.

**3131.** $u = \arcsin \dfrac{x}{z}$, där $z = \sqrt{x^2 + 1}$; $\dfrac{du}{dx} = ?$

**3132.** $z = \tan(3t + 2x^2 - y)$, $x = \dfrac{1}{t}$, $y = \sqrt{t}$; $\dfrac{dz}{dt} = ?$

**3133.** $u = \dfrac{e^{ax}(y - z)}{a^2 + 1}$, $y = a \sin x$, $z = \cos x$; $\dfrac{du}{dx} = ?$

**3134.** $z = \dfrac{xy \arctan(xy + x + y)}{x + y}$; $dz = ?$

**3135.** $z = (x^2 + y^2) e^{\frac{x^2 + y^2}{xy}}$; $\dfrac{\partial z}{\partial x} = ?$ $\dfrac{\partial z}{\partial y} = ?$ $dz = ?$

**3136.** $z = f(x^2 - y^2, e^{xy})$; $\dfrac{\partial z}{\partial x} = ?$ $\dfrac{\partial z}{\partial y} = ?$

**3137.** Visa att funktionen $z = \arctan \dfrac{x}{y}$, där $x = u + v$, $y = u - v$,

Satisfierar ekvationen $\dfrac{\partial z}{\partial u} + \dfrac{\partial z}{\partial v} = \dfrac{u - v}{v^2 + u^2}$.

**3138.** Visa att funktionen $z = \varphi(x^2 + y^2)$, då $\varphi(u)$ —
är deriverbar funktion, satisfierar ekvationen $y \dfrac{\partial z}{\partial x} - x \dfrac{\partial z}{\partial y} = 0$.

**3139.** $u = \sin x + F(\sin y - \sin x)$; Visa att $\dfrac{\partial u}{\partial y} \cos x + \dfrac{\partial u}{\partial x} \cos y = \cos x \cos y$, för varje deriverbar funktione $F$.

**3140.** $z = \dfrac{y}{f(x^2 - y^2)}$; Visa att $\dfrac{1}{x} \dfrac{\partial z}{\partial x} + \dfrac{1}{y} \dfrac{\partial z}{\partial y} = \dfrac{z}{y^2}$, för varje deriverbar funktione $f$.

# FACIT

**3124.** $e^{\sin t - 2t^3} (\cos t - 6t^2)$.    **3125.** $\sin 2t + 2e^{2t} + e^t (\sin t + \cos t)$.

**3126.** $\dfrac{3 - 12t^2}{\sqrt{1 - (3t - 4t^3)^2}}$.    **3127.** $\dfrac{\partial z}{\partial u} = 3u^3 \sin v \cos v (\cos v - \sin v)$;

$$\dfrac{\partial z}{\partial v} = u^3 (\sin v + \cos v)(1 - 3 \sin v \cos v).$$

**3128.** $\dfrac{\partial z}{\partial u} = 2 \dfrac{u}{v^2} \ln (3u - 2v) + \dfrac{3u^2}{v^2 (3u - 2v)}$;

$$\dfrac{\partial z}{\partial v} = - \dfrac{2u^2}{v^3} \ln (3u - 2v) - \dfrac{2u^2}{v^2 (3u - 2v)}.$$

**3129.** $\dfrac{\partial u}{\partial x} = \dfrac{e^x}{e^x + e^y}$;   $\dfrac{du}{dx} = \dfrac{e^x + 3e^{x^3} x^2}{e^x + e^{x^3}}$.

**3130.** $\dfrac{dz}{dx} = \dfrac{e^x (x + 1)}{1 + x^2 e^{2x}}$.    **3131.** $\dfrac{du}{dx} = \dfrac{1}{1 + x^2}$.

**3132.** $\dfrac{dz}{dt} = \left( 3 - \dfrac{4}{t^3} - \dfrac{1}{2\sqrt{t}} \right) \sec^2 \left( 3t + \dfrac{2}{t^2} - \sqrt{t} \right)$.    **3133.** $\dfrac{du}{dx} = e^{ax} \sin x$.

**3134.** $dz = \dfrac{y^2\, dx + x^2\, dy}{(x + y)^2} \operatorname{arctg} (xy + x + y) + \dfrac{xy\, [(y + 1)\, dx + (x + 1)\, dy]}{(x + y)\, [1 + (xy + x + y)^2]}$.

**3135.** $\dfrac{e^{\frac{x^2 + y^2}{xy}}}{x^2 y^2} [(y^4 - x^4 + 2xy^3)\, x\, dy + (x^4 - y^4 + 2x^3 y)\, y\, dx]$.

**3136.** $\dfrac{\partial z}{\partial x} = 2x \dfrac{\partial f}{\partial u} + ye^{xy} \dfrac{\partial f}{\partial v}$,   $\left. \right\}$   $u = x^2 - y^2$;

$$\dfrac{\partial z}{\partial y} = - 2y \dfrac{\partial f}{\partial u} + xe^{xy} \dfrac{\partial f}{\partial v} \quad \left. \right\} \quad v = e^{xy}.$$