Stencil N2 **Partiell derivering**

Beräkna alla partiella derivator av följande funktioner.
Välj funktioner av olika typer och lös gärna 25% av problem.

3039. $z = \frac{u}{v} + \frac{v}{u}$.

3040. $z = \frac{x^3 + y^3}{x^2 + y^3}$.

3041. $z = (5x^2y - y^3 + 7)^3$.

3042. $z = x\sqrt{y} + \frac{y}{\sqrt[3]{x}}$.

3043. $z = \ln(x + \sqrt{x^2 + y^2})$.

3044. $z = \operatorname{arctg} \frac{x}{y}$.

3045. $z = \frac{1}{\operatorname{arctg} \frac{y}{x}}$.

3046. $z = x^y$.

3047. $z = \ln(x^2 + y^2)$.

3048. $z = \ln \frac{\sqrt{x^2 + y^2} - x}{\sqrt{x^2 + y^2} + x}$

3049. $z = \arcsin \frac{\sqrt{x^2 - y^2}}{\sqrt{x^2 + y^2}}$.

3050. $z = \ln \operatorname{tg} \frac{x}{y}$.

3051. $z = e^{-\frac{x}{y}}$.

3052. $z = \ln(x + \ln y)$.

3053. $u = \operatorname{arctg} \frac{v + w}{v - w}$.

3054. $z = \sin \frac{x}{y} \cos \frac{y}{x}$.

3055. $z = \left(\frac{1}{3}\right)^{\frac{y}{x}}$.

3056. $z = (1 + xy)^y$.

3057. $z = xy \ln(x + y)$.

3058. $z = x^{xy}$.

3059. $u = xyz$.

3060. $u = xy + yz + zx$.

3061. $u = \sqrt{x^2 + y^2 + z^2}$.

3062. $u = x^3 + yz^2 + 3yx - x + z$.

3063. $w = xyz + yzv + zvx + vxy$.

3064. $u = e^{x(x^2 + y^2 + z^2)}$.

3065. $u = \sin(x^2 + y^2 + z^2)$.

3066. $u = \ln(x + y + z)$.

3067. $u = x^{\frac{y}{z}}$.

3068. $u = x^{yz}$.

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3039. $\frac{\partial z}{\partial u} = \frac{1}{v} - \frac{v}{u^2}$; $\frac{\partial z}{\partial v} = -\frac{u}{v^2} + \frac{1}{u}$.

3040. $\frac{\partial z}{\partial x} = \frac{x^4 + 3x^2y^2 - 2xy^3}{(x^2 + y^2)^2}$; $\frac{\partial z}{\partial y} = \frac{y^4 + 3x^2y^2 - 2x^3y}{(x^2 + y^2)^2}$.

3941. $\frac{\partial z}{\partial x} = 30xy(5x^2y - y^3 + 7)^2$; $\frac{\partial z}{\partial y} = 3(5x^2y - y^3 + 7)^2(5x^2 - 3y^2)$.

3042. $\frac{\partial z}{\partial x} = \sqrt{y} - \frac{y}{3\sqrt[3]{x^4}}$; $\frac{\partial z}{\partial y} = \frac{x}{2\sqrt{y}} + \frac{1}{\sqrt[3]{x}}$.

3043. $\frac{\partial z}{\partial x} = \frac{1}{\sqrt{x^2 + y^2}}$; $\frac{\partial z}{\partial y} = \frac{y}{x^2 + y^2 + x\sqrt{x^2 + y^2}}$.

3044. $\frac{\partial z}{\partial x} = \frac{y}{x^2 + y^2}$; $\frac{\partial z}{\partial y} = -\frac{x}{x^2 + y^2}$.

3045. $\frac{dz}{dx} = \frac{y}{(x^2 + y^2)\left(\operatorname{arctg} \frac{y}{x}\right)^2}$; $\frac{\partial z}{\partial y} = -\frac{x}{(x^2 + y^2)\left(\operatorname{arctg} \frac{y}{x}\right)^2}$.

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3046. $\frac{\partial z}{\partial x} = yx^{y-1}$; $\frac{\partial z}{\partial y} = x^y \ln x$. 3047. $\frac{\partial z}{\partial x} = \frac{2x}{x^2+y^2}$; $\frac{\partial z}{\partial y} = \frac{2y}{x^2+y^2}$.

3048. $\frac{\partial z}{\partial x} = -\frac{2}{\sqrt{x^2+y^2}}$; $\frac{\partial z}{\partial y} = \frac{2x}{y\sqrt{x^2+y^2}}$.

3049. $\frac{\partial z}{\partial x} = \frac{xy\sqrt{2}}{(x^2+y^2)\sqrt{x^2-y^2}}$; $\frac{\partial z}{\partial y} = -\frac{x^2\sqrt{2}}{(x^2+y^2)\sqrt{x^2-y^2}}$.

3050. $\frac{\partial z}{\partial x} = \frac{2}{y \sin \frac{2x}{y}}$; $\frac{\partial z}{\partial y} = -\frac{2x}{y^2 \sin \frac{2x}{y}}$.

3051. $\frac{\partial z}{\partial x} = -\frac{1}{y} e^{-\frac{x}{y}}$; $\frac{\partial z}{\partial y} = \frac{x}{y^2} e^{-\frac{x}{y}}$.

3052. $\frac{\partial z}{\partial x} = \frac{1}{x+\ln y}$; $\frac{\partial z}{\partial y} = \frac{1}{y(x+\ln y)}$.

3053. $\frac{\partial u}{\partial v} = -\frac{w}{v^2+w^2}$; $\frac{\partial u}{\partial w} = \frac{v}{v^2+w^2}$.

3054. $\frac{\partial z}{\partial x} = \frac{1}{y}\cos\frac{x}{y}\cos\frac{y}{x} + \frac{y}{x^2}\sin\frac{x}{y}\sin\frac{y}{x}$;

$\frac{\partial z}{\partial y} = -\frac{x}{y^2}\cos\frac{x}{y}\cos\frac{y}{x} - \frac{1}{x}\sin\frac{x}{y}\sin\frac{y}{x}$.

3055. $\frac{\partial z}{\partial x} = \frac{y}{x^2} 3^{-\frac{y}{x}} \ln 3$; $\frac{\partial z}{\partial y} = -\frac{1}{x} 3^{-\frac{y}{x}} \ln 3$.

3056. $\frac{\partial z}{\partial x} = y^2(1+xy)^{y-1}$; $\frac{\partial z}{\partial y} = xy(1+xy)^{y-1} + (1+xy)^y \ln(1+xy)$.

3057. $\frac{\partial z}{\partial x} = y\ln(x+y) + \frac{xy}{x+y}$; $\frac{\partial z}{\partial y} = x\ln(x+y) + \frac{xy}{x+y}$.

3058. $\frac{\partial z}{\partial x} = x^{xy}x^{y-1}(y\ln x + 1)$; $\frac{\partial z}{\partial y} = x^y x^{xy}\ln^2 x$.

3059. $\frac{\partial u}{\partial x} = yz$; $\frac{\partial u}{\partial y} = xz$; $\frac{\partial u}{\partial z} = xy$.

3060. $\frac{\partial u}{\partial x} = y+z$; $\frac{\partial u}{\partial y} = x+z$; $\frac{\partial u}{\partial z} = x+y$.

3061. $\frac{\partial u}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}}$; $\frac{\partial u}{\partial y} = \frac{y}{\sqrt{x^2+y^2+z^2}}$; $\frac{\partial u}{\partial z} = \frac{z}{\sqrt{x^2+y^2+z^2}}$.

3062. $\frac{\partial u}{\partial x} = 3x^2+3y-1$; $\frac{\partial u}{\partial y} = z^2+3x$; $\frac{\partial u}{\partial z} = 2yz+1$.

3063. $\frac{\partial w}{\partial x} = yz+vz+vy$; $\frac{\partial w}{\partial y} = xz+zv+vx$.

$\frac{\partial w}{\partial z} = xy+yv+vx$; $\frac{\partial w}{\partial v} = yz+xz+xy$.

3064. $\frac{\partial u}{\partial x} = (3x^2+y^2+z^2)e^{x(x^2+y^2+z^2)}$;

$\frac{\partial u}{\partial y} = 2xye^{x(x^2+y^2+z^2)}$; $\frac{\partial u}{\partial z} = 2xze^{x(x^2+y^2+z^2)}$.

3065. $\frac{\partial u}{\partial x} = 2x \cos(x^2 + y^2 + z^2)$; $\frac{\partial u}{\partial y} = 2y \cos(x^2 + y^2 + z^2)$;

$\frac{\partial u}{\partial z} = 2z \cos(x^2 + y^2 + z^2)$.

3066. $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial u}{\partial z} = \frac{1}{x + y + z}$.

3067. $\frac{\partial u}{\partial x} = \frac{y}{z} x^{\frac{y}{z} - 1}$; $\frac{\partial u}{\partial y} = \frac{1}{z} x^{\frac{y}{z}} \ln x$; $\frac{\partial u}{\partial z} = -\frac{y}{z^2} x^{\frac{y}{z}} \ln x$.

3068. $\frac{\partial u}{\partial x} = y^z x^{y^z - 1}$; $\frac{\partial u}{\partial y} = z y^{z-1} x^{y^z} \ln x$; $\frac{\partial u}{\partial z} = y^z x^{y^z} \ln x \ln y$.

3069. $\frac{2}{5}$, $\frac{1}{5}$.

3070. 0, $\frac{1}{4}$.

3071. $\frac{\partial z}{\partial x} = 2(2x + y)^{2x+y} [1 + \ln(2x + y)]$;

$\frac{\partial z}{\partial y} = (2x + y)^{2x+y} [1 + \ln(2x + y)]$.

3072. $\frac{\partial z}{\partial x} = \frac{3}{x \ln y} \left(1 + \frac{\ln x}{\ln y}\right)^2$; $\frac{\partial z}{\partial y} = -\frac{3 \ln x}{y \ln^2 y} \left(1 + \frac{\ln x}{\ln y}\right)^2$.