

Stencil 5. Partiella derivator av högre ordning

3181. $x = x^3 + xy^2 - 5xy^3 + y^5$. Visa att $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.

3182. $z = x^y$. Visa att $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.

3183. $z = e^x (\cos y + x \sin y)$. Visa att $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.

3184. $z = \operatorname{arctg} \frac{y}{x}$. Visa att $\frac{\partial^2 z}{\partial y^2 \partial x} = \frac{\partial^2 z}{\partial x \partial y^2}$.

3185-92: Beräkna $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial x \partial y}$ och $\frac{\partial^2 z}{\partial y^2}$ för givna funktioner.

3185. $z = \frac{1}{3} \sqrt{(x^2 + y^2)^3}$. 3186. $z = \ln(x + \sqrt{x^2 + y^2})$.

3187. $z = \operatorname{arctg} \frac{x+y}{1-xy}$. 3188. $z = \sin^2(ax + by)$.

3189. $z = e^{xy}$. 3190. $z = \frac{x-y}{x+y}$.

3191. $z = y^{\ln x}$. 3192. $z = \arcsin(xy)$.

3193. $u = \sqrt{x^2 + y^2 + z^2 - 2xz}$; $\frac{\partial^2 u}{\partial y \partial z} = ?$

3194. $z = e^{-xy^2}$; $\frac{\partial^2 z}{\partial x^2 \partial y} = ?$

3195. $z = \ln(x^2 + y^2)$; $\frac{\partial^2 z}{\partial x \partial y^2} = ?$ 3196. $z = \sin xy$; $\frac{\partial^2 z}{\partial x \partial y^2} = ?$

3197. $w = e^{xyz}$; $\frac{\partial^2 w}{\partial x \partial y \partial z} = ?$ 3198. $v = x^m y^n z^p$; $\frac{\partial^6 v}{\partial x \partial y^3 \partial z^3} = ?$

3200. $u = e^x (x \cos y - y \sin y)$. Visa att $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

3201. $u = \ln \frac{1}{\sqrt{x^2 + y^2}}$; Visa att $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

3202. $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$; Visa att $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

3203. $r = \sqrt{x^2 + y^2 + z^2}$; Visa att

$$\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{2}{r}, \quad \frac{\partial^3 (\ln r)}{\partial x^2} + \frac{\partial^3 (\ln r)}{\partial y^2} + \frac{\partial^3 (\ln r)}{\partial z^2} = \frac{1}{r^3}.$$

3204. för vilket värde av konstanten a funktionen $v = x^3 + axy^3$ uppfyller ekvationen $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$?

3205. $z = \frac{y}{y^2 - a^2 x^2}$; Visa att $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

3206. $v = \frac{1}{x-y} + \frac{1}{y-z} + \frac{1}{z-x}$; Visa att

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} + 2 \left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 v}{\partial z \partial x} \right) = 0.$$

3207. $z = f(x, y)$, $\xi = x + y$, $\eta = x - y$. Visa att

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 4 \frac{\partial^2 z}{\partial \xi \partial \eta}.$$

3208. $v = x \ln(x+r) - r$, где $r^2 = x^2 + y^2$. Visa att

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{1}{x+r}.$$

3210. $y = \varphi(x - at) + \psi(x + at)$. Visa att $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$,

för alla C^2 -funktioner φ och ψ .

3211. $u = \varphi(x) + \psi(y) + (x - y)\psi'(y)$.

Visa att $(x - y) \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial u}{\partial y}$

(φ och ψ är C^2 -funktioner)

3212. $z = y\varphi(x^2 - y^2)$. Visa att $\frac{1}{x} \frac{\partial z}{\partial x} + \frac{1}{y} \frac{\partial z}{\partial y} = \frac{z}{y^2}$ (φ är en C^2 -funktion)

3213. $r = x\varphi(x + y) + y\psi(x + y)$. Visa att

$$\frac{\partial^2 r}{\partial x^2} - 2 \frac{\partial^2 r}{\partial x \partial y} + \frac{\partial^2 r}{\partial y^2} = 0$$

(φ och ψ är C^2 -funktioner)

Facit:

3185. $\frac{\partial^2 z}{\partial x^2} = \frac{2x^2 + y^2}{\sqrt{x^2 + y^2}}$; $\frac{\partial^2 z}{\partial y^2} = \frac{x^2 + 2y^2}{\sqrt{x^2 + y^2}}$; $\frac{\partial^2 z}{\partial x \partial y} = \frac{xy}{\sqrt{x^2 + y^2}}$.

3186. $\frac{\partial^2 z}{\partial x^2} = -\frac{x}{(x^2 + y^2)^{\frac{3}{2}}}$; $\frac{\partial^2 z}{\partial y^2} = \frac{x^3 + (x^2 - y^2)\sqrt{x^2 + y^2}}{(x^2 + y^2)^{\frac{3}{2}}(x + \sqrt{x^2 + y^2})^2}$;

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{y}{(x^2 + y^2)^{\frac{3}{2}}}$$

3187. $\frac{\partial^2 z}{\partial x^2} = -\frac{(x^2 + y^2)^{\frac{2}{3}}}{(1 + x^2)^3}$; $\frac{\partial^2 z}{\partial y^2} = -\frac{2y}{(1 + y^2)^2}$; $\frac{\partial^2 z}{\partial x \partial y} = 0$.

3188. $\frac{\partial^2 z}{\partial x^2} = 2a^2 \cos 2(ax + by)$; $\frac{\partial^2 z}{\partial y^2} = 2b^2 \cos 2(ax + by)$;

$$\frac{\partial^2 z}{\partial x \partial y} = 2ab \cos 2(ax + by)$$

3189. $\frac{\partial^2 z}{\partial x^2} = e^{xe^y} + 2y$; $\frac{\partial^2 z}{\partial y^2} = x(1 + xe^y)e^{xe^y} + y$; $\frac{\partial^2 z}{\partial x \partial y} = (1 + xe^y)e^{xe^y} + y$.

3190. $\frac{\partial^2 z}{\partial x^2} = -\frac{4y}{(x + y)^3}$; $\frac{\partial^2 z}{\partial y^2} = \frac{4x}{(x + y)^3}$; $\frac{\partial^2 z}{\partial x \partial y} = \frac{2(x - y)}{(x + y)^3}$.

3191. $\frac{\partial^2 z}{\partial x^2} = \frac{\ln y (\ln y + 1)}{x^2} e^{\ln x \ln y}$; $\frac{\partial^2 z}{\partial y^2} = \frac{\ln x (\ln x - 1)}{y^2} e^{\ln x \ln y}$;

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\ln x \ln y + 1}{xy} e^{\ln x \ln y}$$

3192. $\frac{\partial^2 z}{\partial x^2} = \frac{xy^3}{\sqrt{(1 - x^2y^2)^3}}$; $\frac{\partial^2 z}{\partial y^2} = \frac{x^3y}{\sqrt{(1 - x^2y^2)^3}}$; $\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{\sqrt{(1 - x^2y^2)^3}}$.

3193. $\frac{(x - z)y}{\sqrt{(x^2 + y^2 + z^2 - 2xz)^3}}$. 3194. $2y^3(2 + xy^2)e^{xy^2}$.

3195. $\frac{4x(3y^3 - x^2)}{(x^2 + y^2)^3}$. 3196. $-x(2 \sin xy + xy \cos xy)$.

3197. $(x^2y^2z^2 + 3xyz + 1)e^{xyz}$.

3198. $mn(n - 1)(n - 2)p(p - 1)x^{m-1}y^{n-3}z^{p-2}$. 3204. $a = -3$.