Stencil 5.
Variablebyte, lokala extrempunkter

Utför variabelsubstitutionen. I de fall du kan, lös differentialekvationen.

1. $2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0,\ \xi = x,\ \eta = x - 2y.$

2. $2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = x^2 + 4y^2$, samma substitution som ovan.

3. $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0,\ y > 0,\ \xi = x,\ \eta = x^2 + y^2.$

4. $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} - 6\frac{\partial^2 u}{\partial y^2} = 0,\ \xi = 3x + y,\ \eta = y - 2x.$

5. $x\frac{\partial^2 u}{\partial x \partial y} - y\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} = 0,\ x > 0,\ \xi = x,\ \eta = xy.$

6. $x^2\frac{\partial^2 u}{\partial x^2} - y^2\frac{\partial^2 u}{\partial y^2} = 0,\ x > 0,\ y > 0,\ \xi = xy,\ \eta = \frac{x}{y}.$

Bestäm alla stationära punkter till funktionerna nedan och avgör deras karaktär.

5.1.1 $f(x, y) = x^2 + xy + y^2 - 12x - 3y.$

5.1.2. $f(x, y) = 3 + 2x - y - x^2 + xy - y^2.$

5.1.3. $f(x, y) = 3x + 6y - x^2 - xy + y^2.$

5.1.4. $f(x, y) = 4x^2 - 4xy + y^2 + 4x - 2y + 1.$

5.4.1. $f(x, y) = \frac{x+y}{xy} - xy.$

5.4.2. $f(x, y) = \frac{8}{x} + \frac{x}{y} + y.$

5.6.5. $f(x, y) = \frac{x^3}{3} + 3x^2e^y - e^{-y^2}$

28. $f(x, y) = 2x^3 - 6xy + 3y^2.$

29. $f(x, y) = \frac{x}{y} + \frac{8}{x} - y.$

30. $f(x, y) = xye^{-(x^2+y^2)/2}.$

31. $f(x, y) = x \sin y.$

32. $f(x, y, z) = x^2y + y^2z + z^2 - 2x.$

5.13.2. $f(x, y, z) = 8 - 6x + 4y - 2z - x^2 - y^2 - z^2.$

5.13.4 $f(x, y, z) = x^3 + y^2 + z^2 + 6xy - 4z.$

Svar:

Funktionerna f och g nedan antas vara tillräckligt många gånger kontinuerligt deriverbara, i övrigt godtyckliga. **1.** $\frac{\partial u}{\partial \xi} = 0$, $u(x,y) = f(x-2y)$; **2.** $2\frac{\partial u}{\partial \xi} = \xi^2 + (\xi - \eta)^2$, $u(x,y) = f(x-2y)+\frac{1}{6}x^3+\frac{4}{3}y^3$; **3.** $\frac{\partial u}{\partial xi} = 0$, $u(x,y) = f(x^2+y^2)$; **4.** $\frac{\partial^2 u}{\partial \xi \partial \eta} = 0$, $u(x,y) = f(3x+y)+g(y-2x)$; **5.** $\frac{\partial^2 u}{\partial \xi \partial \eta} = 0$, $u(x,y) = f(xy) + g(x)$; **6.** $\frac{\partial^2 u}{\partial \xi \partial \eta} - \frac{1}{2\xi}\frac{\partial u}{\partial \eta} = 0$, $u(x,y) = f(xy) + xg(x/y)$;

5.1.1. $(7,-2)$, lok. min. = -39; **5.1.2.** $(1,0)$, lok. max. = 4; **5.1.3.** inga extrempunkter; **5.1.4.** inte strngt lok. min. = 0 p linjen $y = 2x+1$; **5.4.1.** $(-1,-1)$, lok. max. = -3 ; **5.4.2.** $(4,2)$, lok. min. = 6 ; **5.6.5.** $(0,0)$, lok. min. = -1 ;

23. $\frac{1}{2}$; **24.** $-\frac{1}{3}$; **25.** $\frac{1}{12}$; **26.** 1; **27.** $\frac{1}{2}$; **28.** $(0,0)$, sadelpkt; $(1,1)$, lok. min; **29.** $(-4,2)$, lok. max; **30.** $(0,0)$, sadelpkt; $(1,1),(-1,-1)$, lok. max; $(1,-1),(-1,1)$, lok. min; **31.** $(0,k\pi)$, $k \in \mathbb{Z}$, sadelpkter; **32.** $(1,1,-\frac{1}{2})$, sadelpkt. **5.13.2.** $(-3,2,-1)$, lok.max. = 22; **5.13.4.** $(6,-18,2)$, lok min.= -112.