Formelblad för analyskurserna i M1 98/99

Trigonometri.

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x)\cos(y) = \frac{1}{2}(\cos(x-y) + \cos(x+y))$$

$$\sin(x)\sin(y) = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$

$$\sin(x)\cos(y) = \frac{1}{2}(\sin(x-y) + \sin(x+y))$$

$$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$$

Integralkatalog

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad a \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \frac{1}{cos^2 x} dx = \tan x + C$$

$$\int \frac{1}{sin^2 x} dx = -\cot x + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \quad 0 < a \neq 1$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a}\arctan\frac{x}{a} + C \quad a \neq 0$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \frac{1}{\sqrt{x^2+a}} dx = \ln|x + \sqrt{x^2+a}| + C \quad a \neq 0$$

$$\int \sqrt{x^2+a} dx = \frac{1}{2}(x\sqrt{x^2+a} + a\ln|x+\sqrt{x^2+a}|) + C$$

$$\int \frac{1}{(x^2+1)^n} dx = I_n$$

$$I_{n+1} = \frac{x}{2n(x^2+1)^n} + \frac{2n-1}{2n} I_n$$

Maclaurinutvecklingar

$$e^x = \sum_{k=0}^{n} \frac{x^k}{k!} + x^{n+1}B(x)$$

$$\sin x = \sum_{k=1}^{n} (-1)^{k-1}\frac{x^{2k-1}}{(2k-1)!} + x^{2n+1}B(x)$$

$$\cos x = \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!} + x^{2n+2}B(x)$$

$$(1+x)^\alpha = \sum_{k=0}^{n} \binom{\alpha}{k} x^k + x^{n+1}B(x) \qquad |x| < 1$$

$$\ln(1+x) = \sum_{k=1}^{n} \frac{(-1)^{k+1}x^k}{k} + x^{n+1}B(x) \qquad -1 < x \leq 1$$

$$\arctan x = \sum_{k=1}^{n} (-1)^{k-1}\frac{x^{2k-1}}{2k-1} + x^{2n+1}B(x) \qquad |x| \leq 1$$

$$\binom{\alpha}{k} = \frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-k+1)}{k(k-1)(k-2)\ldots 1} \qquad \binom{\alpha}{0} = 1$$

Övrigt

Tyngdpunkten (x_T, y_T, z_T) för Ω ges av $x_T = \dfrac{\iiint_\Omega x\rho(x,y,z)\,dxdydz}{\iiint_\Omega \rho(x,y,z)\,dxdydz}$, analogt för y_T, z_T.

$\rho(x,y,z)$ är densiteten.