(c) $$\|u-\pi_h u\|_{L^\infty(I_1)} = \max_{x\in I_1} |u(x)-\pi_h u(x)|$$

$$u(x)-\pi_h u(x) = 1-(x-1)^2-1.5x$$

$$= 0.5x - x^2 = r(x)$$

Maximera $r(x)$ på I_1.

$r'(x) = 0.5-2x = 0$ då $x = 0.25$

$r''(x) = -2 \Rightarrow r$ har lokalt maximum

i $x = 0.25$

$$r(0.25) = 0.125 - 0.0625 = 0.0625$$

Slutsats: $\|u-\pi_h u\|_{L^\infty(I_1)} = 0.0625$

(d) Formeln säger att

$$\|u-\pi_h u\|_{L^\infty(I_1)} \leq \frac{1}{8}(0.5)^2 \max_{x\in I_1} |u''(x)|$$

$$= \frac{1}{8}\cdot\frac{1}{4}\cdot 2 = 0.0625$$

(e) $$\|u-\pi_h u\|^2_{L^2(I_1)} = \int_{I_1} |u-\pi_h u|^2\,dx \leq$$

$$\leq \int_{I_1} (\max|u-\pi_h u|)^2\,dx = |I_1|\,\|u-\pi_h u\|^2_{L^\infty(I_1)}$$

$$\leq |I_1|\,C_1^2 = C_2^2$$