NUMERICAL LINEAR ALGEBRA, 2007

HOMEWORK ASSIGNMENT 3

Well performed this homework assignment gives 1 credit point

To be handed in by October 4 at the latest

Exercise 3 a. Solve the question Q3.4 in the text book. Try to obtain a **symmetric** augmented system. (**0.5 point**)

Exercise 3 b. In the standard least squares method we minimize $\| b - \hat{b} \|_2$ subject to the constraint $\hat{b} \in Span(A)$, that is we seek the closest compatible system $A\hat{x} = \hat{b}$ by varying the right hand side. In the so called **total** least squares method we allow also the matrix to vary, that is we seek the closest compatible system $\hat{A}\hat{x} = \hat{b}$ to the system $Ax = b$ i.e. we minimize $\| [A \; b] - [\hat{A} \; \hat{b}] \|_2$ subject to $\hat{b} \in Span(\hat{A})$. Show that this problem can be solved by SVD factorization of the augmented matrix $[A \; b] = U\Sigma V^T$. Use the fact that the approximating system $\hat{A}\hat{x} = \hat{b}$ must be compatible and show that the solution can be written $\hat{x} = -\frac{1}{v_{n+1,n+1}}[v_{1,n+1} \; v_{2,n+1} \; ... \; v_{n,n+1}]^T$, where $v_{i,j}$ are elements of the matrix V. (**0.5 points**)
Hint: Relations between the SVD factors and the fundamental subspaces of a matrix.

COMPUTER EXERCISE 3

To be handed in by October 4 at the latest

a) Consider the problem of fitting a polynomial of degree d to data points $(x_i, \; y_i), \; i = 1, \; ... \; , \; m$ in the plane. Let the polynomial be written $p(x) = \sum_{j=0}^{d} a_j x^j$. Choose $d = 14, \; a_j = 1, \; j = 0, \; ... \; , \; 14$ and 21 equally spaced points on the interval $[0, \; 1]$. Compute $y_i = p(x_i), \; i = 1, \; ... \; , \; 21$.
Now we try to recover the coefficients a_j by solving the overdetermined system $A\hat{a} = y$, where the columns of A are powers of the vector x. Compare the error $\|a - \hat{a}\|_2$ in the following methods for solving the system of equations:
1. The normal equations
2. QR factorization
3. A regularization method defined like this: QR factorization of the enlarged system $\begin{bmatrix} A \\ \alpha I \end{bmatrix} = QR$ and then $\hat{a} = R \backslash Q^T \begin{bmatrix} y \\ 0 \end{bmatrix}$ with the properly chosen value $\alpha = 10^{-7}$.
4. Truncated least squares with the properly chosen number of 12 terms in SVD.
Draw conclusions. Explain the differences between the methods.

b) Use QR factorization to solve the following problem, a least squares problem with linear constraints: Fit a quadratic polynomial to the points (1,1) (2,2) (4,2) (5,3) (6,3) (7,4) (8,5), and (9,6) with the restriction that the curve has to pass exactly through the points (4,2) and (9,6). Plot points and the fitting curve.

Note: To obtain the highest grade on this exercise you should write efficient codes, for instance without explicit inverses.