

## NUMERICAL LINEAR ALGEBRA, 2007

### HOMEWORK ASSIGNMENT 4

Well performed this homework assignment gives 1 credit point

To be handed in by October 15 at the latest

**Exercise 4 a).** Solve the question Q4.6 in the text book. (0.5 point)

**Exercise 4 b).** Solve the question Q4.8 in the text book. (0.5 point)

### COMPUTER EXERCISE 4

To be handed in by October 15 at the latest

a) Solve the question Q4.15 in the text book. Find the program **qrplot**, written by Jim Demmel and revised by Ivar Gustafsson, on the course web side.

b) In order to compute the eigenvalues of the pentadiagonal matrix  $A =$

$$\begin{bmatrix} 4 & 2 & 1 & 0 & 0 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 \\ 1 & 2 & 4 & 2 & 1 & 0 \\ 0 & 1 & 2 & 4 & 2 & 1 \\ 0 & 0 & 1 & 2 & 4 & 2 \\ 0 & 0 & 0 & 1 & 2 & 4 \end{bmatrix}$$

we at first reduce it to tridiagonal form by the following technique:

(i) Determine a Givens rotation  $R(2, 3, \theta)$  which zeros out the element in position (3,1) in the matrix  $R(2, 3, \theta) A$ . Compute the transformed matrix  $A^{(1)} = R(2, 3, \theta) A R^T(2, 3, \theta)$ .

(ii) In the matrix  $A^{(1)}$  a new nonzero element has been introduced (in the lower part of the matrix). Show how this element can be zeroed out by a new rotation without introducing any new nonzero elements.

(iii) **A theoretical exercise for attaining the highest grade on this computer exercise.** Device a zero chasing algorithm, based on the ideas in (i) and (ii), to reduce a general symmetric pentadiagonal matrix to a symmetric tridiagonal matrix. For a  $n \times n$  matrix,  $n$  an even number, how many rotations are needed? How many floating points operations are required?

**Hint:** Run the program **chasing**, written by two former students in this course and available from the course web side, to see how your method is supposed to work.