NUMERICAL LINEAR ALGEBRA, 2008

HOMEWORK ASSIGNMENT 1

Well performed this homework assignment gives 1 credit point.

To be handed in by September 11 at the latest.

Exercise 1 a. Solve question Q1.7 in the text book. You may restrict the proof to the real case $x, y \in R^n$, i.e. $y^H = y^T$. (**0.5 point**)

Exercise 1 b. Consider the system of equations $(A + \rho uv^T)\hat{x} = b$, where A is a square matrix, u, v, $\hat{x}$, and b are column vectors and ρ is a scalar.
Suppose you already have solved the system of equations $Ax = b$ to get x. Show how you efficiently can get the solution $\hat{x}$ by solving another system **with the same coefficient matrix** A but a new right hand side. What do you gain in efficiency, number of flops, compared to solving the system with the new matrix $A + \rho uv^T$? (**0.5 point**)
Hint: Use the Sherman-Morrisons formula, see question Q2.13 in the text book. You need not prove this formula.

COMPUTER EXERCISE 1, with theoretical parts

To be handed in by September 11 at the latest

Solve question Q1.20 in the text book. You may use the MATLAB program **polyplot**, originally written by Jim Demmel and modified by Ivar Gustafsson, see the link at the course webpage. You must not hand in graphs for all suggested data in Q1.20 part 1. Take the three cases:
$r = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], \quad e = 10^{-8}$
$r = [2, 4, 8, 16, 32, 64, 128, 256, 512, 1024], \quad e = 10^{-3}$
$p = [1, 2, -2, 4, 5, 7], \quad e = 0.1.$
Hint to question 2 of Q1:20: The number $c(i)e$, where
$c(i) = \frac{1}{|\frac{dp}{dx}(r(i))|} \sum_{j=0}^{d} | a_j || r(i) |^j$,
is an upper bound for the change in the root $r(i)$ when the coefficients are perturbed at most e (relatively) that is when $\frac{|\bar{a}_j - a_j|}{|a_j|} \leq e$. In order to prove this statement you need to use some calculus. Also show that a particular perturbation in the case $d = 1$ (polynomial of degree one), almost gives equality in the upper bound, that is $c(i)$ can be taken as the condition number for the problem of determing the roots.
Comment on Q1:20 part 3. You are supposed to both prove the statement and make computations by using **poly** and **roots** in MATLAB to verify the theory.
Hand in computed results for $p = (x - 2)^4$, $\epsilon = 10^{-8}$ and $p = x^2(x - 3)^5$, $\epsilon = 10^{-6}$ and compare with the theory.
The theoretical proof requires some complex calculus. Express the arising m complex roots explicitely.
Note: To recieve the highest grade on this exercise you must **show** the hint to question 2 and **prove** the statement in part 3.