

Department of  
Mathematics  
Göteborg

**EXAMINATION FOR  
NUMERICAL LINEAR ALGEBRA, TMA265  
2008-10-23**

**DATE: Thursday 23 October 2008    TIME: 8.30 - 12.30    PLACE: V**

Examiner:            Ivar Gustafsson, tel: 772 10 94  
Teacher on duty: Ivar Gustafsson  
Solutions:           Will be announced at the end of the exam on the board nearby room MHF21  
Result:              Will be sent to you by November 6 at the latest  
                         Your marked examination can be received at the student's office  
                         at Mathematics Department, daily 12.30-13  
Grades:              To pass requires 13 point, including bonus points from homework assignments  
                         Grades are evaluated by a formula involving also the computer exercises  
Aids:                 None (except dictionaries)

Instructions:

- State your methodology carefully. Motivate your statements clearly.
- Only write on one page of the sheet. Do not use a red pen. Do not answer more than one question per page.
- Sort your solutions by the order of the questions. Mark on the cover the questions you have answered. Count the number of sheets you hand in and fill in the number on the cover.

**GOOD LUCK!**

**Question 1.**

- a) Define the concept: symmetric, indefinite matrix. **(1p)**
- b) Describe a stable symmetric factorization of a symmetric, indefinite matrix. **(2p)**

**Question 2.**

- a) Describe matrix by matrix multiplication;  $C=A*B+C$ , based on blocking. Derive the ratio  $q$  of flops to memory references, when the matrices are  $n$  by  $n$  and the fast memory contains  $M$  words. **(2p)**
- b) Mention two operations being on BLAS-3 level. **(1p)**

**Question 3.**

- a) Define the concept: compact singular value decomposition (SVD) of a matrix. **(1p)**
- b) Use SVD to express the projection of a vector on the row space of a matrix. **(2p)**

**Question 4.**

Perform the first step in the bidiagonalization of a matrix  $A = \begin{bmatrix} 1 & 3 & 0 & 4 \\ -1 & 1 & 1 & -2 \\ 0 & 3 & 1 & 0 \\ -1 & -1 & 1 & -2 \\ 1 & 3 & 2 & 0 \end{bmatrix}$  i.e.

find Householder reflections  $H$  and  $Z$  such that  $A_1 = HAZ$  has the form

$$A_1 = \begin{bmatrix} x & x & 0 & 0 \\ 0 & x & x & x \end{bmatrix}. \quad (3\text{p})$$

**Question 5.**

a) Define the concepts: left and right eigenvectors of a square matrix. (1p)

b) Define the concept: condition number of an eigenvalue. (1p)

c) Determine left and right eigenvectors as well as the condition numbers of the eigenvalues of the Householder matrix  $H = I - 2uu^T$ , where  $u \in R^n$  with  $\|u\|_2 = 1$ . (2p)

**Question 6.**

Rotate the matrix  $\begin{bmatrix} 2 & 4 & 3 \\ 4 & 1 & 0 \\ 3 & 0 & 2 \end{bmatrix}$  to a tridiagonal matrix by a single similar transformation based on a Givens rotation. (3p)

**Question 7.** Prove the following theorem by Bauer-Fike:

Let  $A \in R^{n \times n}$  be diagonalizable:  $X^{-1}AX = D = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$  and let  $\mu$  be an eigenvalue of the matrix  $A + E$ , where  $E \in R^{n \times n}$ .

Then for any p-norm:

$$\min_{1 \leq i \leq n} |\mu - \lambda_i| \leq \kappa_p(X) \|E\|_p$$

where  $\kappa_p$  is the condition number in p-norm. (3p)

**Question 8.**

Describe briefly all steps in the method: Householder implicit QR iteration, for computing eigenvalues of a real unsymmetric square matrix. (3p)