NUMERICAL LINEAR ALGEBRA, 2009

HOMEWORK ASSIGNMENT 4

Well performed this homework assignment gives 1 credit point

To be handed in by October 12 at the latest

Exercise 4 a). Solve question Q4.6 in the text book. (**0.5 point**)

Exercise 4 b). Solve question Q4.8 in the text book. (**0.5 point**)

COMPUTER EXERCISE 4

To be handed in by October 12 at the latest

a) Solve the question Q4.15 in the text book. Find the program **qrplot**, written by Jim Demmel and revised by Ivar Gustafsson, on the course webpage.
Note on grades: For highest grade, carefully answer to all four questions.

b) In order to compute the eigenvalues of the pentadiagonal matrix $A = \begin{bmatrix} 4 & 2 & 1 & 0 & 0 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 \\ 1 & 2 & 4 & 2 & 1 & 0 \\ 0 & 1 & 2 & 4 & 2 & 1 \\ 0 & 0 & 1 & 2 & 4 & 2 \\ 0 & 0 & 0 & 1 & 2 & 4 \end{bmatrix}$ we at first reduce it to tridiagonal form by the following technique:

(i) Determine a Givens rotation $R(2,3,\theta)$ which zeros out the element in position (3,1) in the matrix $R(2,3,\theta)\ A$. Compute the transformed matrix $A^{(1)} = R(2,3,\theta)\ A\ R^T(2,3,\theta)$.

(ii) In the matrix $A^{(1)}$ a new nonzero element has been introduced (in the lower part of the matrix). Show how this element can be zeroed out by a new rotation without introducing any new nonzero elements.

(iii) **A theoretical exercise for attaining the highest grade on this computer exercise.** Device a zero chasing algorithm, based on the ideas in (i) and (ii), to reduce a general symmetric pentadiagonal matrix to a symmetric tridiagonal matrix. For a $n \times n$ matrix, n an even number, how many rotations are needed? How many floating points operations are required?
Hint: Run the program **chasing**, written by two former students in this course and available from the course webpage, to see how your method is supposed to work.