NUMERICAL LINEAR ALGEBRA, 2009

HOMEWORK ASSIGNMENT 5

Well performed this homework assignment gives 1 credit point

To be handed in by October 22 at the latest

Exercise 5 a). Solve question Q5.2 in the text book. Observe that Theorem 3.3, part (4) can be extended to the case of rectangular A, see Q3.14. (**0.5 point**)

Exercise 5 b). Solve question Q5.6 in the text book. (**0.5 point**)

COMPUTER EXERCISE 5

To be handed in by October 22 at the latest

a) Consider the question Q5.13 in the text book. Show that the two algorithm are equivalent by implementing them in MATLAB and comparing the results.

b) Symmetric tridiagonal matrices are interesting. We get them as intermediate results when we compute eigenvalues of full symmetric matrices by the Householder QR method, and we get them from the Lanczos algorithm for large sparse eigenvalue problems in Chapter 7 of the text book.
Here we shall study one step of the divide and conquer algorithm (section 5.3.3) for symmetric tridiagonal matrices i.e. we shall not use it recursively. We will see how the eigenvalues move when a certain parameter α moves from 0 to 1. The case $\alpha = 0$ corresponds to a completely divided matrix and $\alpha = 1$ corresponds to the original matrix.

(i) In a first simple division we replace the element b_m in T, see page 217, with αb_m. Let now α vary from 0 to 1 in small steps and study how the eigenvalues move. We get n paths of eigenvalues. Some of the eigenvalues move, some do not. Do they really cross each other? This question has no cut answer, when two eigenvalues float together they lose their identities and you cannot see whether a path continues up or down, that is cross another, or if it turnes. Use test matrices below and illustrate graphically a case when eigenvalues float together. Size $n = 10$ and division in two equal parts should be possible.

(ii) Let us now do it as described in the book, that is we make rank one perturbations with four matrix elements. Also in this case we introduce a parameter, that is we multiply the rank one matrix by α, running from 0 to 1, and study how the eigenvalues move. In this case we get a monotone behaviour, that is all paths either go up or down depending on the sign of b_m. Illustrate this graphically on the test matrices below.

Test matrices.

1. The matrix of a vibrating guitar string with all diagonal elements 2 and subdiagonal elements -1. (This one is easy to generate, but very slow to converge.)

2. The result of running the MATLAB command T=hess(A+A'), with A a matrix of normally distributed random numbers.

Note: For highest grade on this exercise you must find and hand in a case where at least two eigenvalues float together in case (i) above. You may need several random tests for the matrix T to obtain this situation.