NUMERICAL LINEAR ALGEBRA, 2010

HOMEWORK ASSIGNMENT 1

Well performed this homework assignment gives 1 credit point.

To be handed in by September 9 at the latest.

Exercise 1 a. Solve question Q1.7 in the text book. (**0.5 point**)

Exercise 1 b. Consider the following two systems of linear equations $Ax = b$ and $(A + \rho uv^T)\hat{x} = b$, where A is a given square $n \times n$ -matrix, u, v, and b are given column vectors and ρ is a given scalar. Note that the matrix $A_1 = A + \rho uv^T$ is a rank-one correction of A. Show how you efficiently can get the solution to both systems by just factorizing the matrix A and not A_1. Present the order in which all computations are made. In the case of a large order n, what do you gain in efficiency (number of flops) compared to factorizing both matrices? (**0.5 point**)
Hint: Use the Sherman-Morrisons formula, see question Q2.13 in the text book. You need not prove this formula.

COMPUTER EXERCISE 1, with theoretical parts

To be handed in by September 9 at the latest

Solve question Q1.20 in the text book. You may use the MATLAB program **polyplot**, originally written by Jim Demmel and modified by Ivar Gustafsson, see the link at the course webpage. You must not hand in graphs for all suggested data in Q1.20 part 1. Take the following three cases. Answer the question in this part carefully for the first case and explain the difference in sensitivity for different roots by studying the number $c(i)$ given in the hint below. r stands for roots and p stands for coefficients (p must be set in the program).
$r = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], \quad e = 10^{-8}$
$r = [2, 4, 8, 16, 32, 64, 128, 256, 512, 1024], \quad e = 10^{-3}$
$p = [1, 2, -2, 4, 5, 7], \quad e = 0.1.$
Hint to part 2 of Q1:20: The number $c(i)e$, where
$c(i) = \frac{1}{|\frac{dp}{dx}(r(i))|} \sum_{j=0}^{d} \mid a_j \mid\mid r(i) \mid^j,$
is an upper bound for the change in the root $r(i)$ when the coefficients are perturbed at most e (relatively) that is when $\frac{|\bar{a}_j - a_j|}{|a_j|} \leq e$. In order to prove this statement you need to use some calculus. Also show that a particular perturbation in the case $d = 1$ (polynomial of degree one), almost gives equality in the upper bound, that is $c(i)$ can be taken as the condition number for the problem of determing the roots.
Comment on Q1:20 part 3. You are supposed to both prove the statement and make computations using **poly** and **roots** in MATLAB to verify the theory.
Hand in computed results for $p = (x-2)^4$, $\epsilon = 10^{-8}$ and $p = x^2(x-3)^5$, $\epsilon = 10^{-6}$ and compare with the theory. Note that $q = 1$ and $q = x^2$, respectively in the two cases.
The theoretical proof requires some complex calculus. Express the arising m complex roots explicitely.
Note about grading: To recieve the highest grade (5) on this exercise you must **prove** the hint to question 2 and **prove** the statement in part 3. If you prove one of these you will get grade (4) and if you just do the computations you will get a (3).