NUMERICAL LINEAR ALGEBRA, 2010

## HOMEWORK ASSIGNMENT 4

**Well performed this homework assignment gives 1 credit point**

**To be handed in by October 11 at the latest**

**Exercise 4 a)**. Solve question Q4.2 in the text book. (**0.5 point**)

**Exercise 4 b)**. Solve question Q4.8 in the text book. (**0.5 point**)

## COMPUTER EXERCISE 4

**To be handed in by October 11 at the latest**

**a)** Solve the question Q4.15 in the text book. Find the program **qrplot**, written by Jim Demmel and revised by Ivar Gustafsson, on the course webpage.
**Note on grades:** For highest grade, carefully answer to all four questions.

**b)** In order to compute the eigenvalues of the pentadiagonal matrix $A = \begin{bmatrix} 4 & 2 & 1 & 0 & 0 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 \\ 1 & 2 & 4 & 2 & 1 & 0 \\ 0 & 1 & 2 & 4 & 2 & 1 \\ 0 & 0 & 1 & 2 & 4 & 2 \\ 0 & 0 & 0 & 1 & 2 & 4 \end{bmatrix}$ we at first reduce it to tridiagonal form by the following technique:

(i) Determine a Givens rotation $R(2,3,\theta)$ which zeros out the element in position (3,1) in the matrix $R(2,3,\theta)\ A$. Compute the transformed matrix $A^{(1)} = R(2,3,\theta)\ A\ R^T(2,3,\theta)$.

(ii) In the matrix $A^{(1)}$ a new nonzero element has been introduced (in the lower part of the matrix). Show how this element can be zeroed out by a new rotation without introducing any new nonzero elements.

(iii) **A theoretical exercise for attaining the highest grade on this computer exercise.** Device a zero chasing algorithm, based on the ideas in (i) and (ii), to reduce a general symmetric pentadiagonal matrix to a symmetric tridiagonal matrix. For an $n \times n$ matrix, $n$ an even number, how many rotations are needed? How many floating point operations are required?
**Hint:** Run the program **chasing**, written by two former students in this course and available (in p-code) from the course webpage, to see how your method is supposed to work.