Department of
Mathematics
Göteborg

EXAMINATION FOR
NUMERICAL LINEAR ALGEBRA, TMA265/MMA600
2009-12-02

DATE: Wednesday December 2 TIME: 13 - 17 PLACE: MVL15

Examiner:	Ivar Gustafsson, tel: 772 10 94
Teacher on duty:	Ivar Gustafsson
Solutions:	Will be announced at the end of the exam on the board nearby room MVF21
Result:	Will be sent to you by December 15 at the latest
	Your marked examination can be received at the student's office
	at Mathematics Department, daily 12.30-13
Grades:	To pass requires 13 point, including bonus points from homework assignments
	Grades are evaluated by a formula involving also the computer exercises
Aids:	None (except dictionaries)

Instructions:
- State your methodology carefully. Motivate your statements clearly.
- Only write on one page of the sheet. Do not use a red pen. Do not answer more than one question per page.
- Sort your solutions by the order of the questions. Mark on the cover the questions you have answered. Count the number of sheets you hand in and fill in the number on the cover.

GOOD LUCK!

Question 1.
a) Define the concept **stability of a problem**. **(1p)**
b) Define the concept **stability of an algorithm**. **(1p)**
c) State a result regarding stability of the problem $Ax = b, \quad A \in R^{n\times n}$. **(1p)**

Question 2.
Describe on principle how Gaussian elimination on block form with delayed updating can be carried out with basicly BLAS-3 routines and just a small amount of computations on BLAS-2 level. **(3p)**

Question 3.
a) State a stable factorization of a symmetric indefinite real matrix. **(1p)**
b) Consider a so called generalized eigenproblem $Ax = \lambda Bx$, where A is symmetric and B is symmetric, positive definite. Show how this problem can be transformed to a standard symmetric eigenproblem. **(2p)**
Hint: Cholesky factorization.

Question 4.

Use a Householder reflection to perform a similar transformation of the matrix $\begin{bmatrix} 0 & -1 & 1 \\ 4 & 2 & 0 \\ 3 & 4 & 0 \end{bmatrix}$ to upper Hessenberg form. **(3p)**

Question 5.
a) Let A and B be matrices such that the enlarged matrix $[A\ B]$ is nonsingular and $A^T B = O$. Prove that $[A\ B]^{-1} = \begin{bmatrix} A^+ \\ B^+ \end{bmatrix}$, where A^+ and B^+ are generalized inverses. **(2p)**
b) Let A^+ be the Moore Penrose pseudoinverse of $A \in R^{m \times n},\ \ m \geq n$, with $0 < rank(A) < n$. Prove that $\|A^+ A\|_2 = 1$. **(2p)**

Question 6.
a) Express the Schur form of a matrix $A \in C^{n \times n}$. **(1p)**
b) Let $(\bar{\lambda}, \bar{x})$ be a computed eigenpair of a matrix $A \in R^{n \times n}$. Show that $(\bar{\lambda}, \bar{x})$ is an exact eigenpair of a matrix $A + E$, where $\|E\|_2 = \frac{\|r\|_2}{\|x\|_2}$ and $r = A\bar{x} - \bar{\lambda}\bar{x}$ is the residual. **(2p)**

Question 7.
a) Descibe briefly all steps in the method: Householder implicit shift QR iteration, for computing eigenvalues of a real unsymmetric matrix. **(3p)**

Question 8.
Perform the first step of the classical Jacob'i method for computing the eigenvalues of the symmetric matrix $A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$. **(3p)**