Department of
Mathematics
Göteborg

EXAMINATION FOR
NUMERICAL LINEAR ALGEBRA, TMA265/MMA600
2010-12-17

DATE: Friday December 17 TIME: 13.00 - 17.00 PLACE: MVL14

Examiner:	Ivar Gustafsson, tel: 772 10 94
Teacher on duty:	Ivar Gustafsson
Solutions:	Will be announced at the end of the exam on the board nearby room MVF21
Result:	Will be sent to you by December 22 at the latest Your marked examination can be received at the student's office at Mathematics Department, daily 12.30-13
Grades:	To pass requires 13 point, including bonus points from homework assignments Grades are evaluated by a formula involving also the computer exercises
Aids:	None (except dictionaries)

Instructions:
- State your methodology carefully. Motivate your statements clearly.
- Only write on one page of the sheet. Do not use a red pen. Do not answer more than one question per page.
- Sort your solutions by the order of the questions. Mark on the cover the questions you have answered. Count the number of sheets you hand in and fill in the number on the cover.

GOOD LUCK!

Question 1.
a) Define the concept **symmetric indefinite matrix**. **(1p)**
b) Suppose A is **symmetric, positive definite**. Device a suitable way to compute $x^T A^{-1} x$ for a vector $x \in R^n$. **(2p)**

Question 2.
Describe on principle how **Gaussian elimination on block form with delayed updating** can be carried out with basicly BLAS-3 routines and just a small amount of computations on BLAS-2 level. **(3p)**

Question 3.
Show that $\|A_k - A\|_2 = \sigma_{k+1}$, where A_k is the **truncated SVD-sum with k terms** of $A \in R^{m\times n}$ and σ_{k+1} is the **singular value** number $k+1$ of A. Verify all steps in the proof. **(3p)**

Question 4.
Consider the **Householder matrix** $H = I - 2uu^T$ for a vector u with norm $\|u\|_2 = 1$.
a) Prove that H is orthogonal. **(1p)**
b) Prove that H is a reflection. **(1p)**
c) Perform the first step of a **QR-factorization of the matrix** $\begin{bmatrix} 0 & -1 \\ 4 & 2 \\ 0 & 3 \\ 3 & 4 \end{bmatrix}$. **(2p)**

Question 5.
Consider a **Givens rotation matrix** $R(\theta)$ in two dimensions. Determine left and right eigenvectors as well as the condition numbers of the eigenvalues. **(2p)**

Question 6.
a) Prove that if Q is an orthogonal $n \times n$ matrix then $\|Qx\|_2 = \|x\|_2$ for all $x \in R^n$. **(1p)**
b) Consider the **full rank linear least squares problem**

$$(P) \quad \min_x \|Ax - b\|_2.$$

Assume that you have got a QR-factorization of $A \in R^{m\times n}$, $m > n$. Show how this can be used to determine the solution to (P). **(2p)**

Question 7.
a) Descibe briefly all steps in the method **Householder implicit shift QR iteration** for computing eigenvalues of a real unsymmetric matrix. **(3p)**

Question 8.
a) Perform the first step of the **classical Jacobi's method** for computing the eigenvalues of the symmetric matrix $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$. **(2p)**
b) Scetch, without going into details, the **one-sided Jacobi's method** for computing the SVD of a matrix $G \in R^{m\times n}$. **(2p)**