Department of
Mathematics
Göteborg

EXAMINATION FOR
NUMERICAL LINEAR ALGEBRA, TMA265/MMA600
2008-11-17

DATE: Monday 17 November **TIME: 13.00 - 17.00** **PLACE: MVL15**

Examiner:	Ivar Gustafsson, tel: 772 10 94
Teacher on duty:	Ivar Gustafsson
Solutions:	Will be announced at the end of the exam on the board nearby room MVF21
Result:	Will be sent to you by November 28 at the latest Your marked examination can be received at the student's office at Mathematics Department, daily 12.30-13
Grades:	To pass requires 13 point, including bonus points from homework assignments Grades are evaluated by a formula involving also the computer exercises
Aids:	None (except dictionaries)

Instructions:
- State your methodology carefully. Motivate your statements clearly.
- Only write on one page of the sheet. Do not use a red pen. Do not answer more than one question per page.
- Sort your solutions by the order of the questions. Mark on the cover the questions you have answered. Count the number of sheets you hand in and fill in the number on the cover.

GOOD LUCK!

Question 1.
a) Let $A \in R^{n\times n}$ be a symmetric, positive definite matrix and let $x \in R^n$ be a vector. Device a suitable way to compute $x^T A^{-1} x$. **(2p)**
b) Let $A \in R^{m\times n}$, $m > n$ have rank n. Show how to derive a Cholesky factorization of $A^T A + \alpha I$, where $\alpha > 0$, by a QR-factorization of an enlarged matrix involving A but not $A^T A$. **(2p)**

Question 2.
Describe on principle how Gaussian elimination without pivoting can be carried out by elementary transformations based on outer products. **(3p)**

Question 3.
Consider a least squares problem with equality constraints:

$$\text{(LSE)} \begin{cases} min_x \|Ax - b\|_2 \\ subject\ to\ Bx = d \end{cases},$$

where $A \in R^{m\times n}$, $m > n$, $B \in R^{p\times n}$, $p < n$. Assume that A and B have full rank. Derive a solution formula for (LSE) based on QR-factorization. **(3p)**

Question 4.
a) Consider the Givens' rotation matrix $R(1, 2, \theta)$ of size $n \times n$, $n \geq 3$. Determine left and right eigenvectors and the condition number with respect to eigenvalue computation of this matrix. **(2p)**
b) State a perturbation result for eigenvalues, including the condition number, for a matrix with distinct eigenvalues. **(1p)**

Question 5.
Use a Householder's reflection to perform a similar transformation of the matrix $\begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 1 \\ 4 & -1 & 1 \end{bmatrix}$ to upper Hessenberg form. **(3p)**

Question 6.
a) Define the concepts algebraic and geometric multiplicity of an eigenvalue. Also define the concept defect eigenvalue and give an example of a matrix with a defect eigenvalue. **(2p)**
b) Express the Schur form of an unsymmetric real matrix. **(1p)**

Question 7. Perform the first step of the classical Jacobi's method for eigenvalue computation on the matrix $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ **(3p)**

Question 8.
Descibe a method for computing the SVD of a matrix $G \in R^{m\times n}$, $m \geq n$, based on bidiagonalization and LR-iteration. **(3p)**