Department of
Mathematics
Göteborg

EXAMINATION FOR
NUMERICAL LINEAR ALGEBRA, TMA265
2008-10-23

DATE: Thursday 23 October 2008 TIME: 8.30 - 12.30 PLACE: V

Examiner:	Ivar Gustafsson, tel: 772 10 94
Teacher on duty:	Ivar Gustafsson
Solutions:	Will be announced at the end of the exam on the board nearby room MVF21
Result:	Will be sent to you by November 6 at the latest Your marked examination can be received at the student's office at Mathematics Department, daily 12.30-13
Grades:	To pass requires 13 point, including bonus points from homework assignments Grades are evaluated by a formula involving also the computer exercises
Aids:	None (except dictionaries)

Instructions:
- State your methodology carefully. Motivate your statements clearly.
- Only write on one page of the sheet. Do not use a red pen. Do not answer more than one question per page.
- Sort your solutions by the order of the questions. Mark on the cover the questions you have answered. Count the number of sheets you hand in and fill in the number on the cover.

GOOD LUCK!

Question 1.
a) Define the concept: symmetric, indefinite matrix. **(1p)**
b) Descibe a stable symmetric factorization of a symmetric, indefinite matrix. **(2p)**

Question 2.
a) Describe matrix by matrix multiplication; C=A*B+C, based on blocking. Derive the ratio q of flops to memory references, when the matrices are n by n and the fast memory contains M words. **(2p)**
b) Mention two operations being on BLAS-3 level. **(1p)**

Question 3.
a) Define the concept: compact singular value decomposition (SVD) of a matrix. **(1p)**
b) Use SVD to express the projection of a vector on the rowspace of a matrix. **(2p)**

Question 4.

Perform the first step in the bidiagonalization of a matrix $A = \begin{bmatrix} 1 & 3 & 0 & 4 \\ -1 & 1 & 1 & -2 \\ 0 & 3 & 1 & 0 \\ -1 & -1 & 1 & -2 \\ 1 & 3 & 2 & 0 \end{bmatrix}$ i.e.
find Householder reflections H and Z such that $A_1 = HAZ$ has the form
$A_1 = \begin{bmatrix} x & x & 0 & 0 \\ 0 & x & x & x \\ 0 & x & x & x \\ 0 & x & x & x \\ 0 & x & x & x \end{bmatrix}$. **(3p)**

Question 5.
a) Define the concepts: left and right eigenvectors of a square matrix. **(1p)**
b) Define the concept: condition number of an eigenvalue. **(1p)**
c) Determine left and right eigenvectors as well as the condition numbers of the eigenvalues of the Householder matrix $H = I - 2uu^T$, where $u \in R^n$ with $\|u\|_2 = 1$. **(2p)**

Question 6.

Rotate the matrix $\begin{bmatrix} 2 & 4 & 3 \\ 4 & 1 & 0 \\ 3 & 0 & 2 \end{bmatrix}$ to a tridiagonal matrix by a single similar transformation based on a Givens rotation. **(3p)**

Question 7. Prove the following theorem by Bauer-Fike:
Let $A \in R^{n \times n}$ be diagonalizable: $X^{-1}AX = D = diag(\lambda_1, \lambda_2, ..., \lambda_n)$ and let μ be an eigenvalue of the matrix $A + E$, where $E \in R^{n \times n}$.
Then for any p-norm:

$$\min_{1 \le i \le n} |\mu - \lambda_i| \le \kappa_p(X)\|E\|_p$$

where κ_p is the condition number in p-norm. **(3p)**

Question 8.
Descibe briefly all steps in the method: Householder implicit QR iteration, for computing eigenvalues of a real unsymmetric square matrix. **(3p)**