Department of
Mathematics
Göteborg

EXAMINATION FOR
NUMERICAL LINEAR ALGEBRA, TMA265/MMA600
2011-12-09

DATE: Friday December 9 TIME: 9.00 - 13.00 PLACE: MVL21 at CTH

Examiner:	Ivar Gustafsson, tel: 772 10 94
Teacher on duty:	Ivar Gustafsson
Solutions:	Will be announced at the end of the exam on the board nearby room MVF21
Result:	Will be sent to you by December 30 at the latest Your marked examination can be received at the student's office at Mathematics Department, daily 12.30-13
Grades:	To pass requires 13 point, including bonus points from homework assignments Grades are evaluated by a formula involving also the computer exercises
Aids:	None (except dictionaries)

Instructions:
- State your methodology carefully. Motivate your statements clearly.
- Only write on one page of the sheet. Do not use a red pen. Do not answer more than one question per page.
- Sort your solutions by the order of the questions. Mark on the cover the questions you have answered. Count the number of sheets you hand in and fill in the number on the cover.

GOOD LUCK!

Question 1.

a) Consider solving a linear system $Ax = b$ with full rank and perturbation δb to the right hand side. Show that the corresponding error δx in the solution is bounded by
$\frac{\|\delta x\|}{\|x\|} \leq \kappa(A)\frac{\|\delta b\|}{\|b\|}$, where $\kappa(A)$ is the condition number of A. **(2p)**
b) Define the concept: Condition number of an eigenvalue of a matrix $A \in R^{n\times n}$. **(1p)**
c) Define the concept: Defective eigenvalue of a matrix $A \in R^{n\times n}$. Also give an example of a matrix with a defective eigenvalue. **(1p)**

Question 2.

Describe on principle how Gaussian elimination without pivoting can be carried out by elementary transformations based on outer products. **(3p)**

Question 3.
Consider the (full rank) linear least squares problem

$$(P) \quad \min_x \|Ax - b\|_2.$$

Assume that you have got a QR-decomposition of A. Show how this can be used to determine the solution to (P). Also give the norm of the smallest residual in terms of the QR-decomposition. **(2p)**

Question 4.
a) Define the Moore-Penrose pseudoinverse of $A \in R^{m \times n}, \quad m \geq n$,
with $0 < r = rank(A) < n$. **(1p)**
b) Let A^+ be the Moore-Penrose pseudoinverse. Show that AA^+ and A^+A are symmetric, that $AA^+A = A$, and that $A^+AA^+ = A^+$. **(1p)**
c) Let A^+ be the Moore-Penrose pseudoinvers of a full rank $(r = n)$ matrix A with a compact QR-factorization $A = Q_1R$. Show that $A^+ = R^{-1}Q_1^T$. **(1p)**

Question 5.
Transform the matrix $A = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ to tridiagonal form by similar transformations.
a) Use Givens' rotations. **(2p)**
b) Use Householders's reflections. **(2p)**
c) Use spectrum slicing with shift $\sigma = 5$ to prove that all eigenvalues of A are ≤ 5 and that one eigenvalue is $= 5$. **(2p)**

Question 6.
Let $AX = XB$, where $A \in R^{n \times n}, \; X \in R^{n \times k}$ and $B \in R^{k \times k}$ with $rank(X) = k < n$.
a) Prove that X is a right invariant subspace with respect to A. **(1p)**
b) Prove that the eigenvalues of B are also eigenvalues of A. **(1p)**
c) Explain the concept **deflation** by using a right invariant subspace. **(2p)**

Question 7.
Descibe briefly all steps in the method: Householder implicit QR iteration, for computing eigenvalues of a real unsymmetric square matrix. **(3p)**