Department of
Mathematics
Göteborg

EXAMINATION FOR
NUMERICAL LINEAR ALGEBRA, TMA265/MMA600
2011-10-21

DATE: Friday October 21 TIME: 9.00 - 13.00 PLACE: MVF33 at CTH

Examiner:	Ivar Gustafsson, tel: 772 10 94
Teacher on duty:	Ivar Gustafsson
Solutions:	Will be announced at the end of the exam on the board nearby room MVF21
Result:	Will be sent to you by November 8 at the latest Your marked examination can be received at the student's office at Mathematics Department, daily 12.30-13
Grades:	To pass requires 13 point, including bonus points from homework assignments Grades are evaluated by a formula involving also the computer exercises
Aids:	None (except dictionaries)

Instructions:
- State your methodology carefully. Motivate your statements clearly.
- Only write on one page of the sheet. Do not use a red pen. Do not answer more than one question per page.
- Sort your solutions by the order of the questions. Mark on the cover the questions you have answered. Count the number of sheets you hand in and fill in the number on the cover.

GOOD LUCK!

Question 1.
Let $A \in R^{m\times n}$, $m > n$ have rank n. Show how to derive a Cholesky factorization of $A^TA + \alpha I$, where $\alpha > 0$, by a QR-factorization of an enlarged matrix involving A but not A^TA. **(2p)**

Question 2.
Describe on principle how **Gaussian elimination on block form with delayed updating** can be carried out with basicly BLAS-3 routines and just a small amount of computations on BLAS-2 level. **(3p)**

Question 3.
Consider a least squares problem with equality constraints:

$$\text{(LSE)} \begin{cases} min_x \|Ax - b\|_2 \\ subject\ to\ Bx = d \end{cases},$$

where $A \in R^{m\times n}$, $m > n$, $B \in R^{p\times n}$, $p < n$. Assume that A and B have full rank. Derive a solution formula for (LSE) based on QR-factorization. **(3p)**

Question 4.
a) Define the Moore Penrose pseudoinverse of $A \in R^{m\times n}$, $m \geq n$, with $0 < rank(A) < n$. **(1p)**
b) Let A and B be matrices such that the enlarged matrix $[A\ B]$ is nonsingular and $A^T B = O$. Prove that $[A\ B]^{-1} = \begin{bmatrix} A^+ \\ B^+ \end{bmatrix}$, where A^+ and B^+ are generalized inverses. **(2p)**

Question 5.
a) Define the concept **similar matrices** and prove that similar matrices have the same eigenvalues. **(2p)**
b) Rotate the matrix $\begin{bmatrix} 2 & 4 & 3 \\ 4 & 1 & 0 \\ 3 & 0 & 2 \end{bmatrix}$ to a tridiagonal matrix by a single similar transformation based on a Givens rotation. **(2p)**

Question 6.
a) Define the concept **stability of a problem**. **(1p)**
b) Define the concept **stability of an algorithm**. **(1p)**
c) Let $(\bar{\lambda}, \bar{x})$ be a computed eigenpair of a matrix $A \in R^{n\times n}$. Show that $(\bar{\lambda}, \bar{x})$ is an exact eigenpair of a matrix $A + E$, where $\|E\|_2 = \frac{\|r\|_2}{\|\bar{x}\|_2}$ and $r = A\bar{x} - \bar{\lambda}\bar{x}$ is the residual. **(2p)**

Question 7.
a) Define the **real Schur form** of a real nonsymmetric square matrix. **(1p)**
b) The unshifted QR iteration method converges to real Schur form. How do you compute the possibly complex eigenvalues when the method has converged? **(1p)**
c) If $|\lambda_1| > |\lambda_2| > ... > |\lambda_{n-1}| > |\lambda_n|$ for the spectrum of a nonsymmetric (possibly complex) square matrix, what is then the convergence result of the unshifted QR iteration method? What about complex eigenvalues? **(1p)**

Question 8.
Descibe the divide and conquer method for computing eigenvalues and eigenvectors of a symmetric tridiagonal matrix. Do not go into implementation details. **(3p)**