

# Numerical Linear Algebra

## Computer Labs

# Computer exercise 1 (1 p.)

Solve question Q1.20 in the text book.

Notes: program polyplot.m is also available to download from the homepage of our course.

Matlab code polyplot.m For "Applied Numerical Linear Algebra", Question 1.20 James Demmel, Aug 1995, modified May 1997  
Form the coefficients of a polynomial specified by its roots, and repeatedly add small perturbations to the coefficients, plotting the resulting perturbed roots

Inputs:  $r$  = vector of polynomial roots  $e$  = maximum relative perturbation to make to each coefficient  $m$  = number of random polynomials to generate

Output: Plot of perturbed roots and  
Generate polynomial coefficients

Example: use function **roots** in matlab in order to find polynomial roots. `roots(C)` computes the roots of the polynomial whose coefficients are the elements of the vector `C`. If `C` has  $N+1$  components, the polynomial is

$$p(X) = C(1) * X^N + \dots + C(N) * X + C(N + 1).$$

- 1. Write discretization of the Poisson's equation in two dimensions

$$- a\Delta u(x, y) = f(x, y) \quad (1)$$

on the unit square  $\{(x, y) : 0 < x, y < 1\}$  with boundary conditions  $u = 0$  on the boundary of this square. Here, the coefficient  $a(x, y)$  is such that

$$a(x, y) = 1 + A(\sin(\frac{\pi}{3}x))^2 \cdot (\sin(\pi/3)y)^2 \quad (2)$$

with values of the amplitude  $A = 3, 12, 26$ .

We produce the mesh with the points  $(x_i, y_j)$  such that  $x_i = ih; y_j = jh$  with  $h = 1/(N + 1)$ , where  $N + 1$  is the number of points in  $x$  and  $y$  directions. You should derive the equation

$$a_{i,j}(4u_{i,j} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} - u_{i,j+1}) = h^2 f_{i,j} \quad (3)$$

- Write equation (3) as a single matrix equation in the form  $Au = f$  with explicit entries of  $A$ .

- 2. Implement Algorithm 2.3 of the text book and solve the linear system of equations  $Au = f$  by the Gaussian elimination for the matrix  $A$  of the size  $n = 100, 200, 400, 800$ . Use different right hand side  $f = 1, 50, 100$  to solve (1).

Alternative: solve the above exercise using PETSc (this will give you 5 p.).

## Computer exercise 3 ( 1 p.)

Solve the problem of the fitting a polynomial  $p(x) = \sum_{i=0}^d \alpha_i x^i$  of degree  $d$  to data points  $(x_i, y_i), i = 1, \dots, m$  in the plane by the method of normal equations and QR decomposition (Algorithm 3.1). Choose the degree of polynomial  $d = 14$ , the interval for  $x \in [0, 1]$ , discretize it by  $N$  points and compute discrete values of  $y(x)$  as  $y_i = y(x_i) = p(x_i)$ . Our goal is to recover coefficients  $\alpha_i$  of the polynomial  $p(x) = \sum_{i=0}^d \alpha_i x^i$  by solving the system  $A\alpha = y$  using the method of normal equations and QR decomposition (Algorithm 3.1). Here, columns of the matrix  $A$  are powers of the vector  $x$ . Compare both methods by computing the relative error  $e$

$$e = \frac{\|\alpha - \alpha^*\|_2}{\|\alpha^*\|_2}$$

Here,  $\alpha_j^* = 1$  are the exact values of the computed coefficients  $\alpha_j$ .

## Computer exercise 4 ( 1 p.)

- Solve question Q4.14 of the text book.
- Solve question Q4.15 of the text book.