Numerical Linear Algebra
Homeworks

Notes

- To pass this course you should do two compulsory home assignments before the final exam. Choose 2 of any 5 assignments.
- Homeworks You do personally (not in groups)
- Sent final report with your assignment to my e-mail before deadline (see course page for deadlines for every home assignment).

Homework 1

- Solve question Q1.1 in the text book (0.5 point).
  Let A be an orthogonal matrix. Show that $det(A) = \pm 1$.
  Show that if B also is orthogonal and $det(A) = -det(B)$ then $A + B$ is singular.
- Solve question Q1.4 in the text book (0.5 point).
  A matrix is strictly upper triangular if it is upper triangular with zero diagonal elements. Show that if A is strictly upper triangular and n by n, then $A^n = 0$.

Homework 2 (2 points)

- Write Algorithm 2.2 for the case $n = 3$.
- Using Algorithm 2.2 perform LU factorization of the matrix A

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 8 & 1 \\ 1 & 1 & 16 \end{bmatrix}$$

- Show that the Algorithm 2.3 is analogous to the Algorithm 2.2.
- Using the Algorithm 2.11 (Cholesky algorithm) perform factorization of the same matrix $A = LL^T$.

Homework 3 (1 point)

- Transform the matrix A to the tridiagonal form using Householder reflection. Describe all steps of this transformation.

$$\mathbf{A} = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 6 & 1 \\ 3 & 1 & 7 \end{bmatrix}$$

- Transform the matrix A to the tridiagonal form using Given's rotation. Describe step-by-step this procedure.

$$\mathbf{A} = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 6 & 1 \\ 3 & 1 & 7 \end{bmatrix}$$

Homework 4 (1 point)

Solve question Q3.9 of the text book.
Let A be m by n with SVD $A = U\Sigma V^T$. compute the SVDs of the following matrices in terms of U, Σ, and V:

1. $(A^T A)^{-1}$
2. $(A^T A)^{-1} A^T$
3. $A(A^T A)^{-1}$
4. $A(A^T A)^{-1} A^T$