

Theoretical Questions for the course

Numerical Linear Algebra

TMA265/MMA600

- Perturbation theory for polynomial evaluation. Stability of polynomial evaluation. Condition number of the polynomial.
- Vector and matrix norms. Condition number, relative condition number of the matrix. Perturbation theory for solution of linear system of equations $Ax = b$.
- Sherman-Morrison formula.
- Gaussian elimination.
- Uniqueness of factorization $A = LU$.
- The need of pivoting.
- Improving the accuracy of a solution of system of linear equations $Ax = b$. Equilibration technique for solution $Ax = b$.
- Real symmetric positive definite matrices. Cholesky factorization.
- Band matrices.
- Matrix factorizations that solve the linear least squares problem: normal equations, QR decomposition, SVD decomposition.
- Least squares problems and methods for their solution: the method of normal equations, QR and SVD decomposition. Least squares for classification problems.
- Orthogonal matrices. Householder transformations.
- Givens rotations.
- QR-factorization by Householder transformation and Givens rotation.
- Tridiagonalization of matrix by Householder transformation and Givens rotation.
- Rank-deficient least squares problems.
- Moore- Penrose pseudoinverse A^+ .
- Solving rank-deficient least squares problems using QR with pivoting.
- Nonsymmetric eigenvalues problems. Jordan and Shur Canonical forms.
- Computing eigenvectors from Shur form.
- Gerschgorin's theorem, Bauer-Fike theorem.
- Algorithms for the nonsymmetric eigenproblems: power method, inverse iteration, QR iteration, Hessenberg reduction, tridiagonal and bidiagonal reduction.
- Regular matrix pencils.
- Algorithms for symmetric eigenproblems: Tridiagonal QR iteration, Rayleigh quotient iteration, Divide-and-conquer algorithm. QR iteration with Wilkinson's shift, bisection and inverse iteration algorithms, Jacobi's method.
- Algorithms for the SVD: QR iteration, LR iteration, divide-and-conquer, bisection and inverse iteration, Jacobi's method for the SVD, one-sided Jacobi.