Numerical Linear Algebra
Homeworks

Notes

- To pass this course you should do two compulsory home assignments before the final exam. Choose any 2 of 4 assignments.
- Homeworks You do personally (not in groups)
- Sent final report with your assignment to my e-mail before deadline (see course page for deadlines for every home assignment). Handwritten home assignments can be left in the red box located beside my office.

Homework 1

- Let A be an orthogonal matrix. Show that $det(A) = \pm 1$. Show that if B also is orthogonal and $det(A) = -det(B)$ then $A + B$ is singular. **(0.5 b.p.)**
- A matrix is strictly upper triangular if it is upper triangular with zero diagonal elements. Show that if A is strictly upper triangular and has size n by n, then $A^n = 0$. **(0.5 b.p.)**

- 1. **(0.5 b.p.)** Write step-by-step (by hand) factorization of the matrix A of size 3×3 with elements a_{ij} using following Algorithm:

  Algorithm 1: LU factorization with pivoting

```
for i=1 to n
     apply permutations of A so a_ii ≠ 0 (permute L, U)
          for example, swap rows j and i of A and of L
          where |a_ji| is the largest entry in |A(i : n, i|;
     /* compute column i of L */
     for j=i+1 to n
          l_ji = a_ji / a_ii
     end for
     /* compute row i of U */
     for j=i to n
          u_ij = a_ij
     end for
```

```
    /* update A_22 */
    for j=i+1 to n
        for k=i+1 to n
            a_jk = a_jk - l_ji * u_ik
        end for
    end for
end for
```

Homework 2

- 2. **(0.5 b.p.)** Using Algorithm 1 write step-by-step (by hand) LU factorization of the following matrix A. Note: compute all intermediate values of L and U matrices and present them too. Don't forget present final L and U matrices such that $A = LU$.

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 8 & 1 \\ 1 & 1 & 16 \end{bmatrix}$$

- 3. **(0.5 b.p.)** On the example of the matrix A of size 3×3 with elements a_{ij} explain why the Algorithm 1 is analogous to the following Algorithm 2:

```
Algorithm 2: LU factorization, overwriting L and U on A:
for i=1 to n
     apply permutations of A (see Algorithm 1)
     for j=i+1 to n
          a_ji = a_ji / a_ii
     end for
     for j=i+1 to n
          for k=i+1 to n
               a_jk = a_jk - a_ji * a_ik
          end for
     end for
end for
```

- 4. **(0.5 b.p.)** Using the Cholesky Algorithm write step-by-step (by hand) Cholesky factorization $A = LL^T$ of the same as in item 2 matrix A:

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 8 & 1 \\ 1 & 1 & 16 \end{bmatrix}$$

Cholesky Algorithm

for j = 1 to n
$l_{jj} = (a_{jj} - \sum_{k=1}^{j-1} l_{jk}^2)^{1/2}$
for i = j + 1 to n
$l_{ij} = (a_{ij} - \sum_{k=1}^{j-1} l_{ik} l_{jk})/l_{jj}$
end for
end for

Homework 3

- **(0.5 b.p.)** Transform the matrix A to the tridiagonal form using Householder reflection. Describe all steps of this transformation.
$$\mathbf{A} = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 6 & 1 \\ 3 & 1 & 7 \end{bmatrix}$$
- **(0.5 b.p.)** Transform the matrix A to the tridiagonal form using Given's rotation. Describe step-by-step this procedure.
$$\mathbf{A} = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 6 & 1 \\ 3 & 1 & 7 \end{bmatrix}$$

Homework 4 **(1 b.p.)**

Let A be m by n with SVD $A = U\Sigma V^T$. Compute following expressions in terms of $U, \Sigma,$ and V:

1. $(A^T A)^{-1}$
2. $(A^T A)^{-1} A^T$
3. $A(A^T A)^{-1}$
4. $A(A^T A)^{-1} A^T$