Numerical Linear Algebra
Homeworks

Notes

- To pass this course you should do **2 compulsory home assignments** before the final exam. Choose any 2 of 4 assignments.
- Homeworks You should do personally (not in groups).
- Download your homeworks in CANVAS or sent pdf file with your assignment to my e-mail *larisa@chalmers.se* before deadline (see the course page for deadlines for every home assignment). Handwritten home assignments can be left in the red box located beside my office.

Homework 1

- Let A be an orthogonal matrix. Show that $det(A) = \pm 1$. Show that if B also is orthogonal and $det(A) = -det(B)$ then $A + B$ is singular. **(0.5 b.p.)**
- A matrix is strictly upper triangular if it is upper triangular with zero diagonal elements. Show that if A is strictly upper triangular and has size n by n, then $A^n = 0$. **(0.5 b.p.)**

Homework 2

- 1. **(0.5 b.p.)**
  Let matrices $A, B \in \mathbb{R}^{n \times n}$. Show that $(AB)^T = B^T A^T$ as well as $(AB)^{-1} = B^{-1} A^{-1}$ (here, A and B are nonsingular).
- 2. **(0.5 b.p.)** Let $A \in \mathbb{R}^{n \times n}$ is s.p.d. matrix. Using Cholesky factorization of A show that $\|x\|_A = (x^T A x)^{1/2}$ defines vector norm.
- 3. **(0.5 b.p.)** Let $D = \text{diag}(d_1, ..., d_n) \in \mathbb{R}^{n \times n}$ is the diagonal matrix such that $d_i \neq 0$. Compute condition numbers $\kappa_1(D), \kappa_2(D), \kappa_\infty(D)$.

- 4. **(0.5 b.p.)** Use the Cholesky Algorithm to decide for which α the following matrix is positive definite:

$$A = \begin{bmatrix} \alpha & 2 \\ 2 & 4 \end{bmatrix}$$

Cholesky Algorithm

for j = 1 to n
$l_{jj} = (a_{jj} - \sum_{k=1}^{j-1} l_{jk}^2)^{1/2}$
for i = j + 1 to n
$l_{ij} = (a_{ij} - \sum_{k=1}^{j-1} l_{ik} l_{jk}) / l_{jj}$
end for
end for

Homework 3

- **(0.5 b.p.)** Transform the matrix A to the tridiagonal form using Householder reflection. Describe all steps of this transformation.
$$\mathbf{A} = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 6 & 1 \\ 3 & 1 & 7 \end{bmatrix}$$
- **(0.5 b.p.)** Transform the matrix A to the tridiagonal form using Given's rotation. Describe step-by-step this procedure.

$$\mathbf{A} = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 6 & 1 \\ 3 & 1 & 7 \end{bmatrix}$$

Homework 4 **(1 b.p.)**

Let $A \in \mathbb{R}^{m \times n}$ with SVD of it: $A = U \Sigma V^T$. Compute following expressions in terms of U, Σ, and V:

1. $(A^T A)^{-1}$
2. $(A^T A)^{-1} A^T$
3. $A(A^T A)^{-1}$
4. $A(A^T A)^{-1} A^T$