Machine learning algorithms for inverse problems
Course Project

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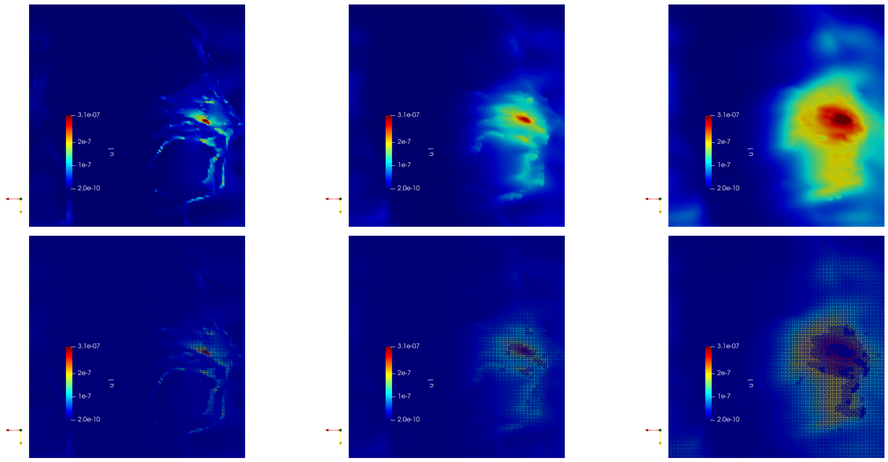
Instructions

- You can work in groups by 2 persons.
- Sent final report for every computer assignment with description of your work together with Matlab or C++/PETSc programs to my e-mail before deadline. Report should have description of used techniques, tables and figures confirming your investigations. Analysis of obtained results is necessary to present in section "Numerical examples" and summarized results - in section "Conclusions". You can download latex or pdf-template for report from the course homepage.
- Information and downloading of PETSc is available at cite [4].
- Matlab and C++ programs for examples in the book [2] are available for download from the course homepage: go to the link of the book [2] and click to "GitHub Page with MATLAB® Source Codes" on the bottom of this page, or copy the link below:

  `https://github.com/springer-math/Numerical_Linear_Algebra_Theory_and_Applications`
- The C++/PETSc code for solution of Helmholtz equation in 2-D with complex exact solution is available for download at `waves24.com`.
- The C++/PETSc code for solution of Helmholtz equation with different preconditioners for solution of the resulting linear system of equations (Jacobi, Gauss-Seidel, SOR) is available for download at `waves24.com`.



coarse mesh | 1 time refined | 2 times refined

Figure 0.1: Reconstructions obtained via AFEM in the transverse plane with the regularization term $\frac{\lambda}{2}\|u\|_2^2$.

Course Project

Regularized adaptive algorithms for detection of tumours in microwave medical imaging

For Master's students: this project can be viewed as beginning for the Master's project "Regularized adaptive algorithms for detection of tumours in microwave medical imaging". Contact me by e-mail for additional information..

In this project we will study different regularization strategies and adaptive algorithms for detection of tumours using microwaves. This problem is a typical Coefficient Inverse Problem (CIP) for determination of complex dielectric permittivity function in Helmholtz equation from scattered electric field in frequency domain. Alternatively, the dielectric permittivity function can be determined from the solution of a Fredholm integral equation of the first kind which is an ill-posed problem. To solve this CIP governed by Helmholtz or by an integral equation, an adaptive finite element method was developed in a C++/PETSc software package WavES [5]. The goal of the current project is further development of mathematical methods presented in the recent paper [1] and implemented in the existing software package WavES [5] for real-life applications in microwave medical imaging. This project is the joint work with the group of Biomedical Imaging at the Department of Electrical Engineering at CTH, Chalmers.

More precisely, in this project students will:

- Study different regularized formulations of the reconstruction problem presented in the paper [1] which can be downloaded from the link

  `https://doi.org/10.1515/jiip-2020-0102`

- Determine the dielectric permittivity function by solving the regularized linear system of equations (LSE) on locally adaptivelly refined meshes using different iterative methods implemented in WavES (Jacobi's method, Gauss-Seidel method, Successive Overrelaxation method (SOR), Conjugate Gradient method, Preconditioned Conjugate Gradient method).

- Test different regularization strategies for choosing the regularization parameter as well as for choosing the regularization terms.
- Study the Lagrangian method and formulate conjugate gradient algorithm for iterative update of the complex dielectric permittivity function.

References

[1] M. G. Aram, L. Beilina, H. Dobsicek Trefna, Microwave Thermometry with Potential Application in Non-invasive Monitoring of Hyperthermia, *Journal of Inverse and Ill-posed problems*, 2020. `https://doi.org/10.1515/jiip-2020-0102`

[2] L. Beilina, E. Karchevskii, M. Karchevskii, Numerical Linear Algebra: Theory and Applications, Springer, 2017.

[3] L. Beilina, M. V. Klibanov, *Approximate global convergence and adaptivity for coefficient inverse problems*, Springer, 2012.

[4] Software package PETSc, `https://www.mcs.anl.gov/petsc/`

[5] Software Package WavES, `waves24.com`