Numerical Linear Algebra
TMA265/MMA600
Computer exercise 3:
Regularized Least Squares and Machine Learning Algorithms for Classification Problem

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Computer exercise 3 (2 b.p.)

Regularized Least squares and machine learning algorithms for classification problem

This exercise can be viewed as beginning for the Master's project "Classification of skin cancer using regularized neural networks" for the skin images from the ISIC project, see link

`https://www.isic-archive.com/#!/topWithHeader/wideContentTop/main`

In this exercise we will study regularized versions of least squares and perceptron learning algorithms for solution of classification problem. Tikhonov's regularization techniques are presented in the paper *Numerical analysis of least squares and perceptron learning for classification problems* which can be downloaded from the link

`https://arxiv.org/pdf/2004.01138.pdf`

Details about AI algorithms for classification together with machine learning techniques for choosing the reg.parameter can be found in [2, 3, 4].

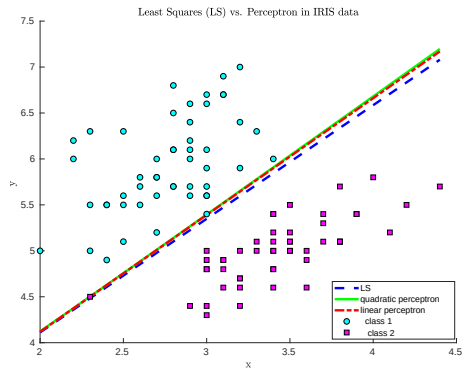

Figure 0.1: Decision lines computed by least squares and the perceptron learning algorithm for separation of two classes using Iris dataset. The dataset iris.csv is available for download from the course page.

Computer exercise 3

Implement in MATLAB all these classification algorithms and present decision lines for following training sets:

- I) Classify IRIS flower data set into several classes using regularized versions of least squares and perceptron learning algorithms. The dataset can be downloaded from the link:

  `https://en.wikipedia.org/wiki/Iris_flower_data_set`

- II) Try to test different regularization techniques (1-2 techniques is enough to test, choose any Tikhonovs technique or machine learning technique) for choosing the regularization parameter. More precisely, test some of Tikhonov's techniques (a priori rule, Morozov's discrepancy, balancing principle) described in Section 5 in the paper *Numerical analysis of least squares and perceptron learning for classification problems* which can be downloaded from the link

  `https://arxiv.org/pdf/2004.01138.pdf`

  Machine learning techniques for choosing the regularization parameter are presented in Section 7 of [3], see `http://www.deeplearningbook.org`.

- III) Optional (not necessary): compute missclassification rate E using the formula (see [4], p. 211-214):

$$\mathrm{E} = \frac{\sum_{i=1}^{K} N_{F,i}}{\sum_{i=1}^{K}(N_{T,i} + N_{F,i})}, \tag{0.1}$$

  where K is the number of classes, $N_{T,i}$ is the number of points of the class i which are classified correctly, $N_{F,i}$ is the number of points of the class i which are classified wrong. Precision for class i can be computed as

$$\mathrm{P}(i) = \frac{N_{T,i}}{N_{T,i} + N_{F,j}}. \tag{0.2}$$

- IV) Optional (not necessary, but if you want to obtain 2 b.p. for this comp.lab.): take some experimental data for classification from the link

  `https://archive.ics.uci.edu/ml/datasets.html`

  or the link for skin images:

  `https://www.isic-archive.com/#!/topWithHeader/wideContentTop/main`

  and classify them using regularized versions of least squares and perceptron learning algorithms.

Try answer to the following questions:

- Analyze different proposed techniques for choosing the regularization parameter. Which one of techniques works best ?
- Analyze what happens with performance of perceptron learning algorithm if we take different learning rates $\eta \in (0, 1]$? For what values of η perceptron learning algorithm is more sensitive and when the iterative process is too slow?
- Analyze which one of the studied classification algorithms perform best and why?
- Try to explain in which case the perceptron algorithm will fail to separate data.

References

[1] L. Beilina, E. Karchevskii, M. Karchevskii, Numerical Linear Algebra: Theory and Applications, Springer, 2017.

[2] Christopher M. Bishop, *Pattern recognition and machine learning*, Springer, 2009.

[3] Ian Goodfellow, Yoshua Bengio and Aaron Courville, *Deep Learning*, MIT Press, 2016, `http://www.deeplearningbook.org`

[4] Miroslav Kurbat, *An Introduction to Machine Learning*, Springer, 2017.