Financial derivative and stochastic analysis, fall 2005

TMA285 or MAM695

Home assignment I

Dead-line November 21, 1.15 pm

1. Exercise 2.9.

2. Suppose you play a sequence of rounds of the game *rock, paper, scissors.* Let the random variables Y_i be defined as

$$Y_i = \begin{cases} 1 & \text{if you win round } i \\ 0 & \text{if there is a draw in round } i \\ -1 & \text{if you loose round } i. \end{cases}$$

Furthermore, let $X_n = \sum_{i=1}^{n} Y_i$.

(a) If both you and your opponent randomly (and independently) with equal probability (i.e. $\frac{1}{3}$) choose rock, paper, or scissors each round, compute $E(X_n)$, $V(X_n)$. What is the distribution of X_n?

(b) What is $E[X_3|\sigma(X_2)]$?

(c) Let now

$$Y_i' = \begin{cases} 2 & \text{if you win with scissors} \\ 1 & \text{if you win rock or paper} \\ 0 & \text{if there is a draw} \\ -1 & \text{if you loose with rock or paper} \\ -2 & \text{if you loose with scissors.} \end{cases}$$

Let $Z_n = \sum_{i=1}^{n} Y_i'$. Compute $E(Z_n)$, $V(Z_n)$, illustrate (e.g. plot) the distribution of Z_n.

(d) If you are working in pairs, play this game for a while (keep track of the accumulated points) and discuss your different thoughts and strategies. Let A_n be the first players score after n rounds. Plot A_i. Does A_i differ from a random sample of Z_n in any significant sense?

3. Let

$$dX = -X^3 \cdot \cos(t)\, dt + dW,$$

where W is a normal Brownian motion, and where $X(0) = 0$.

(a) Give (numerical) estimates of the mean, and the variance of $X(t)$ for $0 < t < 0.3$.

(b) Give an illustration of $Cov(X(t), X(s))$, for $0 < t, s < 0.3$.

Are your results reasonable?

You may work in pairs. Note that this assignment is not compulsory, but you can gain up to 1.5 bonus points to add to your final score.

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