# Financial derivative and stochastic analysis, fall 2005

TMA285 or MAM695

## Home assignment II

**Dead-line December 7, 10.00 am**

If more data is needed for any of the following problems, please either find it yourself, or assume something suitable, but be careful about to note what you assume, or what additional facts you are using.

1. (a) Assume that the Volvo B stock can be modelled using a geometric Brownian motion with constant volatility $\sigma$ and drift $\alpha$. Use the data in the file given on the home-page. to find estimates for $\sigma$ and $\alpha$.

   (b) Use the Black-Scholes-Merton formula to give an expression for the value of a call option with strike price $K$ and exercise date $T$ and where $t = 0$ on November 22.

   (c) Compare this with the data given in the html-file found below the previous link at the home page: Any comments?

   (d) Assume that $\alpha$ is the constant given above, but $\sigma$ is varying and you use a generalized geometric Brownian motion model for the Ericsson B stock. What would be your estimates for $\alpha$ and $\sigma(t)$?

   (e) Can you even estimated both $\alpha(t)$ and $\sigma(t)$? How?

   (f) Suppose you bought 10000 call options for the right price according to the estimated $\sigma(0)$ (and $\alpha(0)$) above, at 2004-11-21 with mature date 2005-11-21 and with strike price 300 SEK. What would that price be?

   (g) Let us now hedge that call option by updating a portfolio with the stock. Plot the stock position $\Delta(t)$ using the historical data.

   (h) Do the same thing, but update only once a week. What will be the difference? Can you come up with a way to measure the efficiency of such a hedging depending on how often the portfolio is updated?

2. (a) Exercise 5.4 with the correction in the first statement of the differential $dS(t)$
$$dS(t) = S(t)\big(r(t)\,dt + \sigma(t)\,d\tilde{W}(t)\big).$$

   (b) Apply the formula derived in the previous exercise for $c(0, S(0))$, and compare with real European call prices for a few different time intervals. Use the $\sigma(t)$ you estimated above, and a suitable function for $r(t)$ (motivate your choice of $r(t)$).

*You may work in pairs. Note that this assignment is not compulsory, but you can gain up to 1.5 bonus points to add to your final score.*

*Torbjörn Lundh, torbjrn@math.chalmers.se, 22/11/05.*