FINANCIAL DERIVATIVES AND STOCHASTIC ANALYSIS (CTH[TMA285]&GU[MAM695]), Period 2, autumn 2006

ASSIGNMENTS

Must be handed in at the latest Friday, November 24 at 14^{45}

Problem 1 Suppose $X : \Omega \to]0, \infty[$ is a bounded random variable. a) Show that

$$E[X \ln X] \geq E[X] \ln E[X].$$

b) What does this inequality say if X is has a uniform distribution in the interval $]0, 1[$?

Problem 2 Suppose the random variable U has a uniform distribution in the interval $\left]-\frac{1}{2}, \frac{1}{2}\right[$ and set $X = \tan(\pi U)$. a) Find the cumulative distribution function of X. b) Find $\mu_X(\left[-1, \sqrt{3}/2\right])$.

Problem 3 Let $n \in \mathbf{N}_+$ and suppose the stochastic process $(V_n(t))_{0 \leq t \leq 1}$, has continuous sample paths, which are affine in each subinterval $\frac{k-1}{n} \leq t \leq \frac{k}{n}$, $k = 1, ..., n$. Moreover, assume $V_n(0) = 0$ and

$$V_n(\frac{k}{n}) = \frac{1}{n^\alpha} \sum_{j=1}^{k} X_j,\ k = 1, ..., n$$

where $(X_j)_{j=1}^n$ is an i.i.d. and $\alpha \in \mathbf{R}_+$. Use MATLAB to draw a picture of a realization of the process $(V_n(t))_{0 \leq t \leq 1}$, if $n = 1000$,

a) $\alpha = 1/2$, and $P[X_1 = 1] = P[X_1 = -1] = \frac{1}{2}$.

b) $\alpha = 1/2$, and $X_1 \in N(0, 1)$.

c) $\alpha = 1$, and X_1 has the same distribution as the random variable X in Problem 2.

Problem 4 Show that the process $e^{-\frac{t}{2}} \cosh W(t)$, $t \geq 0$, is a martingale.

Problem 5 Suppose $T > 0$ and $M(T) = \max_{0 \leq t \leq T} W(t)$. Find $E\left[e^{M(T)}\right]$.