# FINANCIAL DERIVATIVES AND STOCHASTIC ANALYSIS (7.5 hp) (CTH[tma285], GU[MMA710])

Period 2, autumn 2008

Lectures during weeks 44-50 (56 hours): MVH11: Monday  $10^{00}$ - $11^{45}$  (except week 44); Wednesday  $13^{15}$ -15; Friday  $13^{15}$ -15; MVH12: Tuesday  $13^{15}$ -15 (except week 46). Moreover, November 14, 10- $11^{45}$ ,MVF26 November 28, 10- $11^{45}$ ,MVH11

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**Textbook:** S. Shreve, Stochastic Calculus for Finance II. Continuous-Time Models, Springer 2004.

## CONTENTS

#### Week 44

Chapters 1 and 2, Probability Theory: Random variable; Expectation; Change of Measure;  $\sigma$ -algebra; Conditional Expectation. Exercises: 1.5, 1.6, 1.7, 1.8, 1.10, 1.11, 1.15

#### Week 45

**Chapter 3, Brownian Motion:** Symmetric random walk; Brownian Motion; Martingale; Quadratic Variation; Markov Property; First Passage Time Distribution.

Exercises: 2.3, 2.4, 2.5, 2.7

Week 46

Chapter 4, Stochastic Calculus: Itô integral; Itô-Doeblin formula; Black-Scholes-Merton equation; Option Value; Greeks; Put-Call Parity.

**Exercises:** 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 4.4

### Week 47

Chapter 4, Stochastic Calculus: Multivariable Stochastic Calculus; Gaussian Processes.

Chapter 5, Risk-Neutral Pricing: Risk-Neutral Measure; Girsanov's Theorem.

**Exercises:** 4.2, 4.5, 4.6, 4.7, 4.11

### Week 48

Chapter 5, Risk-Neutral Pricing: Martingale Representation. Fundamental Theorems of Asset Pricing; Forward and Futures contracts.
Exercises: 4.13, 4.15, 4.16, 4.19, 5.2, 5.3, 5.4, 5.9, 5.10

#### Week 49

Chapter 6, Connections with Partial Differential Equations: Feynman-Kac Theorem; Interest Rate Models.

Chapter 7, Exotic Options: Barrier Options. Asian Options. Exercises: 6.1, 6.3, 6.4, 6.6

#### Week 50

Chapter 9, Change of Numéraire: Foreign and Domestic Risk-Neutral Measure. Forward Measure.

**Exercises:** 6.7, 6.8, 6.9, 9.5

## EXAMINATION

# Written examinations (4 hours):

December 16, 2008, morning, v April 17, 2009, morning, v

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August 25, 2009, morning v No aids.

The test comprises 15 points; to pass requires at least 6 points (at GU 11 points or more is graded VG; at Chalmers a result greater than or equal to 9 points and smaller than 12 points is graded 4 and a result greater than or equal to 12 points is graded 5).

Assignments

A number of exercises solved and handed in by the student at the latest, November 28 at  $15^{00}$  will result in a maximum of 1 point at the final examination in December 2008.

The written examination thus comprises 15 points, where at least 6 points are of theoretic nature. At least 3 points from the theoretic part are collected from the following list:

(a) Theorem 3.3.4 (b) Theorem 3.4.3 (c) Theorem 3.5.1

(d) Theorem 3.6.1 (e) Theorem 3.6.2 (f) Theorem 3.7.1

(g) Theorem 3.7.3. (h) Theorem 4.2.1 (i) Theorem 4.2.2

(j) Theorem 4.2.3 (k) Theorem 4.4.1 (l) Lemma 4.4.4

(m) Theorem 4.4.9 (n) Lemma 5.2.1 (o) Lemma 5.2.2

(p) Theorem 5.2.3 (q) Theorem 5.4.7 (r) Theorem 6.4.1

(s) Theorem 6.4.3 (t) Theorem 7.2.1 (u) Corollary 7.2.2

(v) Theorem 9.2.1 (w) Theorem 9.2.2

Göteborg October 30, 2008 Christer Borell