**FINANCIAL DERIVATIVES AND STOCHASTIC ANALYSIS (CTH[$tma285$], GU[$MMA710$]), Period 2, 2011**

**ASSIGNMENTS**
Must be handed in at the latest Friday, November 25 at $15^{00}$.

The problems numbered 1-5 are of standard type and you may hand in handwritten solutions. The MATLAB figures in Problems 6 and 7 must be simple to understand and with defined label and axes. No parts of the MATLAB figures are allowed to be written by hand. Good luck!

**Problem 1** Suppose $(X, Y)$ is a centred Gaussian random vector in the plane such that

$$\begin{cases} \sigma_X = \sqrt{E(X^2)} > 0 \\ \sigma_Y = \sqrt{E(Y^2)} > 0 \end{cases}$$

and let $\rho = \frac{E[XY]}{\sigma_X \sigma_Y}$. Find the conditional expectation $E\left[Y^2 \mid X\right]$.

**Problem 2** A stock price process $(S(t))_{0 \leq t \leq T}$ is governed by a geometric Brownian motion. Find the distribution of the random variable

$$\frac{1}{T} \int_0^T \ln S(t) dt.$$

**Problem 3** Suppose $X : \Omega \to ]0, \infty[$ is a bounded random variable. a) Show that

$$E\left[X \ln X\right] \geq E\left[X\right] \ln E\left[X\right].$$

b) Compute $E\left[X \ln X\right]$ and $E\left[X\right] \ln E\left[X\right]$ when $X$ has a uniform distribution in the interval $]0, 1[$.

**Problem 4** Show that the process $e^{-\frac{t}{2}} \cosh W(t)$, $t \geq 0$, is a martingale.

**Problem 5** Suppose $T > 0$ and $M(T) = \max_{0 \le t \le T} W(t)$. Find $E\left[e^{M(T)}\right]$.

**Problem 6** (MATLAB; title('Problem 6'), legend('5 log-Brownian asset price processes')) Consider a stock price process $(S(t))_{0 \le t \le T}$, where

$$S(t) = S(0)e^{(\mu - \frac{\sigma^2}{2})t + \sigma W(t)}.$$

Plot 5 realizations of the stock price process in the same figure if $S(0) = 1$, $\mu = 0.05$, $\sigma = 0.5$, and $T = 1$.

**Problem 7** (MATLAB; title('Problem 7'), legend('Theoretic value','Actual value') Consider a stock price process $(S(t))_{0 \le t \le T}$, where

$$S(t) = S(0)e^{(\mu - \frac{\sigma^2}{2})t + \sigma W(t)}.$$

Suppose $h = T/N$ and $t_n = nh$, $n = 0, ..., N$. Define $cash(t_0)$ by the equation

$$c(t_0, S(t_0), K, T) = \Delta_C(t_0)S(t_0) + cash(t_0)$$

where $\Delta_C(t) = c'_s(t, S(t), K, T)$ and set

$$\Pi_0 = \Delta_C(t_0)S(t_0) + cash(t_0).$$

Next define

$$\Pi_1 = \Delta_C(t_0)S(t_1) + cash(t_0)e^{rh}$$

where $r$ is the interest rate and introduce the quantity $cash(t_1)$ by the equation

$$\Pi_1 = \Delta_C(t_1)S(t_1) + cash(t_1).$$

Let

$$\Pi_2 = \Delta_C(t_1)S(t_2) + cash(t_1)e^{rh}$$

and continue the process until maturity $T$. Plot the theoretic call prices $c(t_i, S(t_i), K, T)$, $i = 0, ..., N$, and the portfolio values $\Pi_i$, $i = 0, ..., N$ in the same figure when $S(t_0) = 100$, $K = 105$, $T = 0.5$, $\mu = 0.05$, $r = 0.03$, $\sigma = 0.35$, and $N = 130$. Finally, find $\Pi_N - (S(T) - K)^+$.