FINANCIAL DERIVATIVES AND STOCHASTIC ANALYSIS (CTH[$tma285$], GU[$MMA710$]), Period 2, 2012

ASSIGNMENTS

Please hand in the problems below at the latest Wednesday, November 21 at 15^{00}.

Problem 1 Suppose $X, Y \in N(0,1)$ are independent. Find $E[X \mid X+Y]$ and $E[XY \mid X+Y]$.

Problem 2 Suppose $T > 0$. A stock price process $(S(t))_{0\le t\le T}$ is governed by a geometric Brownian motion. Find the distribution of the random variable

$$\frac{1}{T}\int_0^T \ln S(t)dt.$$

Problem 3 Suppose $T > 0$. A stock price process $(S(t))_{0\le t\le T}$ is governed by a geometric Brownian motion. Find

$$\lim_{x\to+\infty} x^{-2} \ln P[\ln S(T) > x].$$

Problem 4 Suppose $(\mathcal{F}(t))_{t\ge 0}$ is a filtration for a standard Brownian motion W. If $X(t) = \cosh W(t)$, $t \ge 0$, find

$$E\left[X^2(t) \mid \mathcal{F}(s)\right] \text{ if } s \le t.$$

Problem 5 Let W be a standard Brownian motion and suppose $0 < a < b < \infty$. (a) Find

$$P\left[W(a) < 0 \text{ and } \max_{a\le t\le b} W(t) > 0\right].$$

(b) Find the probability that W has a zero in the interval $[a, b]$.