Partial Differential Equations, TMA372/MMG800

Credit allocation 7.5 hp (Swedish credits).

Description This is the first course on partial differential equations (PDEs) with applications i scinces and engineering. The objective is two-fold:
(i) to cover an up-to-date basic theory and
(ii) to introduce some modern approximation tools.

In the theoretical part, we discuss well-posedness *(existence/uniqueness)* based on weak formulation and minimization problem (Lax-Milgram and Riesz) leading to the existence of a unique solution for the considered problem and study stability concept for the basic PDEs (Poisson, heat, wave and convection-diffusion equations) in the form of Dirichlet, Neumann and Robin (initial) boundary value problems. To solve time dependent PDEs, we also need to consider the study of initial value problems, such as population dynamics and dynamical systems, in the realm of the ordinary differential equations (ODEs).

In the approximation part we focus on constructing and analysing Galerkin finite element methods (approximation by piecewise polynomials, in one and several dimensions) from two points of view. On one hand we analyse the approximation proceedure, and based on both continuous and discrete weak formulations we can gaurantee the existence of a unique discrete solution and its stability. The convergence analysis is based on interpolation estimates and studied both as *a priori* (solution dependent) and *a posteriori* (residual dependent) error estimates.

On the other hand we deal with the implementation aspects of *a priori* and *a posteriori* error bounds. We derive, e.g. stiffness-, mass- and convection matrices, and load vector and eventually reach a linear system of equations to solve numerically. The students are encouraged to use a posteriori error analysis to obtain optimal mesh configurations for concrete problems.

The course is for advanced undergraduates, master programe students in relevant disciplines and graduate (PhD) students. The course consists of 35 lecture hourse, 21 exercise hours.

Prerequisites
The participant is presumed to have
(i) a solid background in calculus of one and several variables,
(ii) knowledge of linear algebra/geometry such as vector and matrix algebra and linear spaces,
(iii) knowledge of the elementary theory of linear ordinary differential equations,
(iv) an acquantaince with the complex number system and the complex exponential function,
(v) a solid background in Fourier analysis (especially method of separation of variables).

Aims and objectives
This course gives an introduction to the modern theory of partial differential equations (PDEs) with applications in science and engineering. It also presernts an introduction to the finite element method as a general tool for numerical solution of PDEs. Iteration procedures and interpolation techniques are also employed to derive *a priori* and *a posteriori* error estimates.

Language: English

Main course literature: An Introduction to the Finite Element Methods (FEM) for Differential Equations, M. Asadzadeh (Lecture Notes).

Reference literature:
• Computational Differential Equations, K. Eriksson, D. Estep, P. Hansbo and C. Johnson, Studentlitteratur 1996.
• The Mathematical Theory of Finite Element Methods, S. Brenner and R. Scott, Springer, ed 3, 2008.

Teaching staff
Examiner and lecturer: Mohammad Asadzadeh, E-mail: mohammad@chalmers.se
TA and Tutor: Announced annually.

Assessment: Home and computer assignments combined with written exam.